MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXIII. MARCH 14, 1913. No. 5

Major E. H. HILLS, C.M.G., F.R.S., PRESIDENT, in the Chair.

Herbert Francis Acocks, Fernyhurst, Ashtead, Surrey; and Arthur Dallas, 4 Peking Road, Shanghai, China,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :—

Captain Francis Henry Wadsworth, R.M.S. *Caledonia*, Anchor Line, Glasgow (proposed by Commr. M. H. Clarke.)

One hundred presents were announced as having been received since the last Ordinary meeting, including, amongst others :—

Astrographic Chart; sixty-eight charts from photographs taken at Algiers, Bordeaux, Paris, and Toulouse, presented by the French Minister of Public Instruction; and twenty charts presented by the San Fernando Observatory.

The Distribution in Space of the Stars in Carrington's Circumpolar Catalogue. By F. W. Dyson, LL.D., Astronomer Royal.

1. If the law of distribution of the linear velocities of the stars in any part of the sky were known, it should be possible by comparison with the observed angular velocities or proper motions to derive the law of distribution of stars according to distance. The velocity distribution of the stars in one co-ordinate has been assumed by Mr. Eddington and Professor Schwarzschild in their researches to be of the form $\frac{h}{\sqrt{\pi}} . e^{-h^2v^2}$. This is the most natural law to assume, and on general grounds it may be expected to hold.

The hypothesis is made that, apart from systematic motions, the proportion of stars with component velocity between $+v$ and $+v+dv$ is

$$\frac{h}{\sqrt{\pi}} . e^{-h^2v^2} . dv \quad . \quad . \quad . \quad . \quad (1)$$

2. If the proportion of stars between the distance r and $r+dr$ be $f\,(r)\,.\,dr$, then the number of these with velocities between $+v$ and $+v+dv$ is

$$\frac{h}{\sqrt{\pi}} . e^{-h^2v^2} . dv . f(r) . dr .$$

If τ be the angular velocity or the proper motion corresponding to v, then $v = r\tau$, and the proportion of stars between the distances r and $r+dr$, whose proper motions (resolved in one direction and apart from systematic motion) are between τ and $\tau+d\tau$, is

$$d\tau \frac{h}{\sqrt{\pi}} r . e^{-h^2r^2\tau^2} . f(r) . dr .$$

If the form of $f\,(r)$ were known, this expression could be integrated between the limits 0 and ∞ of r, and the proportion of stars whose proper motions are between τ and $\tau+d\tau$ would be given as a function of τ.

3. Assume that $f\,(r)$ is of the form $Ar^\lambda . e^{-k^2r^2}$. . . (2)

This form of $f\,(r)$ makes the function vanish at the origin and at infinity; and further, as there are two disposable constants λ and k, it should be possible by suitably choosing them to obtain a satisfactory agreement with observations.

A is chosen so that

$$A \int_0^\infty r^\lambda . e^{-k^2r^2} dr = 1 ,$$

Thus
$$A = \frac{2k^{\lambda+1}}{\Gamma\left(\frac{\lambda}{2}+\frac{1}{2}\right)} \quad . \quad . \quad . \quad . \quad (3)$$

4. The number of proper motions between τ and $\tau + d\tau$.

$$= d\tau \cdot \frac{h}{\sqrt{\pi}} \cdot \frac{2k^{\lambda+1}}{\Gamma\left(\frac{\lambda}{2}+\frac{1}{2}\right)} \cdot \int_0^\infty r^{\lambda+1} \cdot e^{-(h^2\tau^2+k^2)r^2} \cdot dr,$$

$$= \frac{2}{\Gamma\left(\frac{\lambda}{2}+\frac{1}{2}\right)} \cdot \frac{hd\tau}{\sqrt{\pi}} \cdot \frac{k^{\lambda+1}}{(h^2\tau^2+k^2)^{\frac{\lambda}{2}+1}} \cdot \int_0^\infty \tfrac{1}{2} z^{\frac{\lambda}{2}} \cdot e^{-z} dz,$$

$$= \frac{\Gamma\left(\frac{\lambda}{2}+1\right)}{\Gamma\left(\frac{\lambda}{2}+\frac{1}{2}\right)} \cdot \frac{hk^{\lambda+1} \cdot d\tau}{\sqrt{\pi}(h^2\tau^2+k^2)^{\frac{\lambda}{2}+1}},$$

$$= C \frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-\frac{\lambda}{2}-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $$a = \frac{k}{h} \text{ and } C = \frac{1}{\sqrt{\pi}} \cdot \frac{\Gamma\left(\frac{\lambda}{2}+1\right)}{\Gamma\left(\frac{\lambda}{2}+\frac{1}{2}\right)} \quad . \quad . \quad . \quad (5)$$

and it is easily verified that

$$C\int_{-\infty}^{\infty} \frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-\frac{\lambda}{2}-1} . = 1.$$

5. The number of proper motions greater than τ given by this formula is

$$C\int_{\tau}^{\infty} \frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-\frac{\lambda}{2}-1}.$$

When τ is large compared with a

this expression $$= C\int_{\tau}^{\infty} \frac{a^{\lambda+1}}{\tau^{\lambda+2}} \cdot d\tau,$$

$$= \frac{C}{\lambda+1} \cdot \left(\frac{a}{\tau}\right)^{\lambda+1}.$$

Thus the ratio of the number of proper motions greater than τ_1 to that greater than τ_2 (if both τ_1 and τ_2 are large compared with a)

$$= \left(\frac{\tau_2}{\tau_1}\right)^{\lambda+1}.$$

Comparing with the observations it is seen than λ is nearly equal to 1. The values $\lambda = 1{\cdot}2$, $1{\cdot}0$ and $0{\cdot}8$ have been examined in detail. They give for the distribution of proper motions

$$1{\cdot}060 \frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-1{\cdot}6},$$

$$\frac{d\tau}{a} \cdot \left(1+\frac{\tau^2}{a^2}\right)^{-1{\cdot}5},$$

and $$0{\cdot}937 \frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-1{\cdot}4}.$$

6. These formulæ have been compared with the proper motions of the stars contained in Carrington's Circumpolar Catalogue of 3735 stars for the epoch 1855·0. This catalogue extends to a distance of 9° from the North Pole and contains practically all the stars found in the *Bonn Durchmusterung* in this area. It thus reaches a limit of magnitude considerably lower than is given in the catalogues of the *Astronomische Gesellschaft*.

The proper motions have been determined by comparison with the Greenwich Astrographic Catalogue. A careful examination of Carrington's Catalogue is given by Mr. Thackeray in the *Monthly Notices*, vol. lxvi. pp. 320–323. The corrections there found for the declinations have been used. In addition magnitude equation has been determined for each zone and applied to the right ascensions given by Carrington. A further correction of $+''\cdot 28 \cos \alpha$ has been applied to the right ascensions and of $+''\cdot 11 \sin \alpha$ to the declinations. These corrections have been derived by a comparison with Boss's Catalogue, and with such additional stars as are found in Groombridge's Catalogue. These corrections were confirmed by a preliminary discussion of the proper motions of the right ascensions and declinations for each degree of declination.

The elimination of the magnitude equation is important in the consideration of the accidental distribution of the proper motions. The determination did not present any serious difficulty. The periodic corrections are not so satisfactorily determined. They introduce some uncertainty into the value of the parallactic motion used later in the paper; but do not otherwise seriously affect the results.

7. Examination of Carrington's ledgers showed that the probable error of a catalogue place was approximately $\pm''\cdot 40$, and was practically the same in right ascension and declination. The probable error of the positions given in the Greenwich Astrographic Catalogue is $\pm''\cdot 15$ in both co-ordinates. As there is an interval of 45 years between the two catalogues $\pm 1''\cdot 0$ per century was adopted as the probable error of a proper motion in both co-ordinates.

8. The proper motions in right ascension and declination were resolved in the two directions from the Pole towards 0^h of right ascension and towards 6^h of right ascension. The direction towards 0^h is perpendicular to the direction of the Solar Motion, and approximately perpendicular to the directions of the two star streams on Mr. Eddington's hypothesis. On Professor Schwarzschild's hypothesis, all that is necessary is that the direction should be perpendicular to that of the Solar Motion. The linear velocities of the stars in this direction may be regarded as free from a systematic drift, and such that the proportion of stars with velocities between v and $v + dv$ is $\frac{h}{\sqrt{\pi}} \cdot e^{-h^2 v^2} \cdot dv$.

9. Before comparing the observed distribution with the theoretical distribution of proper motions, it was necessary to

correct one or the other for the effect of the accidental probable error of $\pm 1''{\cdot}0$. The changes in the theoretical distributions

$$1{\cdot}060\ \frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-1{\cdot}6} \text{ for } a=1''{\cdot}6,$$

$$\frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-1{\cdot}5} \text{ for } a=1''{\cdot}4 \text{ and } 1''{\cdot}333,$$

and $$0{\cdot}937\ \frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-1{\cdot}4} \text{ for } a=1''{\cdot}2,$$

were determined numerically. The values of the above expressions were calculated for intervals of $0''{\cdot}2$ in $d\tau$, and each of the numbers so obtained was distributed according to the law of errors. Thus, taking the formula $1{\cdot}060\left\{1+\frac{\tau^2}{(1{\cdot}6)^2}\right\}^{-1{\cdot}6}.d\tau$, the number 66·24/1000 values of τ between $\pm 0''{\cdot}1$ were distributed as follows:—

$0''{\cdot}0-0''{\cdot}4$	14·13
$0''{\cdot}4-0''{\cdot}8$	13·12
$0''{\cdot}8-1''{\cdot}2$	11·72
$1''{\cdot}2-1''{\cdot}6$	9·14 etc.

Similarly, the numbers between $+0''{\cdot}1$ and $+0''{\cdot}3$, $-0''{\cdot}1$ and $-0''{\cdot}3$, etc., were distributed into sections of width $0''{\cdot}4$ from $0''{\cdot}0$ to $5''{\cdot}2$. In addition, the numbers $>5''{\cdot}0$, $6''{\cdot}0$, $7''{\cdot}0$ were calculated.

For greater values of τ, I have used a very beautiful formula which Mr. Eddington gave me. The distribution $u(\tau)$ is changed by the error ϵ into $v(\tau)$ where

$$v(\tau)=e^{\frac{D^2}{4h^2}}.u(\tau)$$

where h is the measure of precision corresponding to the probable error ϵ. For the probable error $1''{\cdot}0$ per century adopted

$$v(\tau)=u(\tau)+1{\cdot}1\frac{d^2u}{d\tau^2}+\frac{(1{\cdot}1)^2}{2\,!}\cdot\frac{d^4u}{d\tau^4}+\text{etc.}$$

By calculating $u(\tau)$ for the values

$$\tau=5''{\cdot}0,\ 6''{\cdot}0,\ 7''{\cdot}0, \text{ etc.},$$

and using fourth differences, the values of $v(\tau)$ were found for the higher values of τ.

10. The actual distribution of the proper motions as compared with that given by the various formulæ $\frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-\frac{\lambda}{2}-1}$ is shown in the following table:—

Limits of Centennial Proper Motion.	Number per thousand of stars given by				
	Observation.	$a=1\cdot6$ $\lambda=1\cdot2$	$a=1\cdot4$ $\lambda=1\cdot0$	$a=1\cdot2$ $\lambda=0\cdot8$	$a=1\cdot333$ $\lambda=1\cdot0$
″0·0 – ″0·4	170·6	151·7	154·9	158·0	157·6
0·4 – 0·8	149·5	145·3	148·0	150·7	150·4
0·8 – 1·2	135·5	133·4	135·4	137·3	137·3
1·2 – 1·6	118·5	117·9	118·8	119·4	119·7
1·6 – 2·0	100·2	99·7	99·7	99·7	100·2
2·0 – 2·4	75·7	81·6	81·2	80·8	81·4
2·4 – 2·8	57·9	64·4	63·2	62·0	62·8
2·8 – 3·2	43·6	49·6	48·1	46·6	47·4
3·2 – 3·6	32·4	37·3	35·8	34·4	35·0
3·6 – 4·0	25·3	27·6	26·3	25·0	25·4
4·0 – 4·4	17·4	20·7	19·3	18·0	18·4
4·4 – 4·8	13·7	14·9	14·0	13·1	13·3
4·8 – 5·2	8·9	11·0	10·4	9·1	9·7
5·2 – 6·0	11·6	14·0	13·5	12·8	12·3
6·0 – 7·0	10·3	10·3	9·8	9·3	8·8
7·0 – 8·0	7·8	6·4	5·8	5·6	5·3
8·0 – 10·0	8·7	6·7	6·4	7·1	5·8
10·0 – 15·0	6·8	4·8	6·0	6·1	5·5
15·0 – 20·0	2·2	1·7	1·9	2·2	1·8
>20·0	2·2	1·8	2·5	3·3	2·2
	998·7	1000·9	1000·6	1000·5	1000·5

Comparison of these figures shows that all the formulæ give too small a proportion of stars with proper motions between ″0·0 and ″0·4 a century, and also too small a proportion greater than ″6·0 a century. The best solution is given by $a=1''\cdot2$ and $\lambda=0\cdot8$ except that it gives too large a number $>20''$ a century, 3·3 per thousand giving 12 in the whole catalogue whereas there are actually only 8. Otherwise this fits extremely well; it gives too many proper motions between ″2·0 and ″3·6 a century and too few between ″0·0 and ″0·4. The solution $a=1\cdot4$, $\lambda=1\cdot0$ is better for the very large proper motions giving 9 instead of 8 greater than ″20·0 per century and slightly worse from ″0·0 to ″0·4 and ″2·0 to ″4·0.

11. Both these solutions fit the observations well, and it will be shown later that the differences given by them in the space and velocity distribution are small.

12. But as examination of these residuals shows that they would be further reduced if the probable error of the proper motions were diminished, a more extensive examination of Carrington's ledgers was made. The mean residuals for 1494 stars, each of which had been observed three times, was found to be ″·646 for the right ascensions and ″·556 for the North Polar

distances, or for the mean of the two ″·601. This gives for the probable error of a star place ±″·359. Taking the probable error of a proper motion to be 0″·90, and altering a to the value 1″·47, the accordance of the formula $\frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-\frac{3}{2}}$ with the observations is as shown by the following table:—

Limits of Proper Motion.	Number of Stars.		
	Observed.	Calculated.	Difference.
″ ″			
0·0 – 0·4	634	604	+30
0·4 – 0·8	555	575	−20
0·8 – 1·2	503	517	−14
1·2 – 1·6	440	443	− 3
1·6 – 2·0	372	366	+ 6
2·0 – 2·4	281	288	− 6
2·4 – 2·8	215	224	− 9
2·8 – 3·2	162	163	− 1
3·2 – 3·6	121	120	+ 1
3·6 – 4·0	94	88	+ 6
4·0 – 4·4	64	64	0
4·4 – 4·8	51	47	+ 4
4·8 – 6·0	76	83	− 9
6·0 – 8·0	66	59	+ 9
8·0 – 10·0	33	26	+ 7
10·0 – 15·0	25	24	+ 1
>15″·0	16	18	− 2
...	3708	3708	...

In the diagram (fig. 1) the distribution $\frac{d\tau}{1\cdot4}\left(1+\frac{\tau^2}{1\cdot4^2}\right)^{-\frac{3}{2}}$ is shown by the curve A. The same distribution corrected for an accidental error 1″·0 per century is shown by B. The error curve corresponding to a probable error of 1″·0 per century is shown by C. The distribution $\frac{d\tau}{1\cdot47}\left(1+\frac{\tau^2}{1\cdot47^2}\right)^{-\frac{3}{2}}$ is shown by the curve D. The observations are indicated by the dots.

It will be seen that D fits the observations perfectly: and the probable error computed simply from the ″·36 given by Carrington's ledgers and the ″·15 of the Greenwich results, divided by the difference of epoch 44·5 years is a little less than ″·90 per century.

The probable error of Carrington derived from the discordances in his ledgers probably needs to be slightly increased by the inclusion of a quasi-systematic error, so that the probable error of

a position depending on n observations would be $\sqrt{\epsilon_0^2 + \frac{\epsilon^2}{n}}$ where $\epsilon = ''\cdot 36$. The value of the probable errors of the proper motions will be slightly increased.

As more than 700 of the stars depend on four observations,

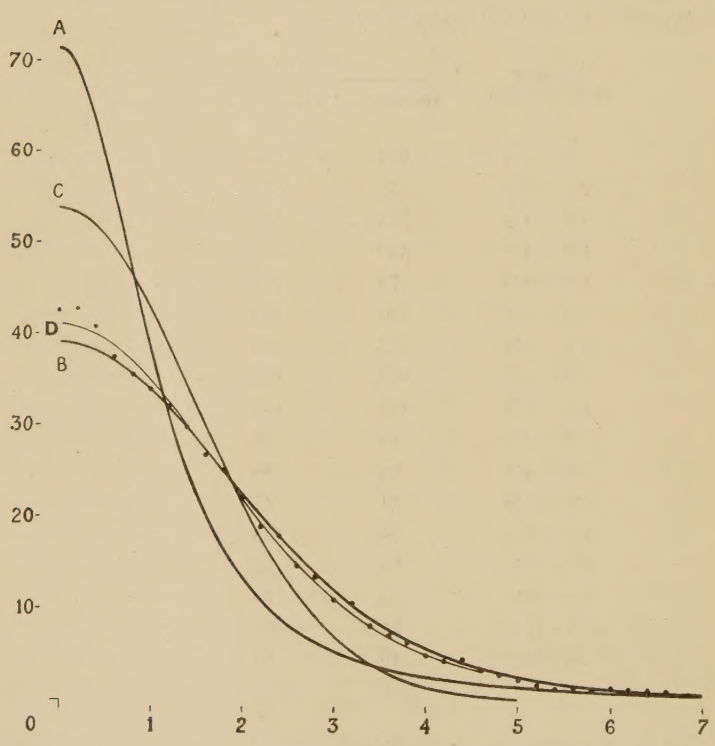


FIG. 1.—DISTRIBUTION OF CROSS PROPER MOTIONS.

The dots are smoothed Observed Values.

The curve **A** shows distribution $d\tau\left(1+\frac{\tau^2}{a^2}\right)^{-1\cdot 5}$ where $a = 1''\cdot 4$.

The curve **B** shows this distribution when a probable error of $1''\cdot 0$ a century is allowed for.

The curve **C** is the Error Curve (probable error $1''\cdot 0$) given for comparison.

The curve **D** is based on the assumption that the probable error of a proper motion is $0''\cdot 90$ a century.

which would reduce the probable error of a catalogue place for these stars from $''\cdot 36$ to $''\cdot 30$, it seems to me that the probable error may be taken as between $0''\cdot 90$ and $1''\cdot 00$ per century. I think that the most probable result may be taken as giving $\frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-\frac{3}{2}}$ for the proportion of proper motions between τ and

$\tau + d\tau$, where $a = 1''{\cdot}44$. This gives for the large proper motions, as compared with the observed numbers, the following :—

Limits of Proper Motion.	Number of Stars.		
	Calculated.	Observed.	Difference.
$5''-6''$	65	55	+10
6 – 8	59	66	− 7
8 – 10	24	33	− 9
$>10''$	40	41	− 1

For the smaller proper motions the distribution is between the two curves B and D. The agreement with observation is excellent, the greatest discordances being 551 stars instead of 588 between $2''{\cdot}0$ and $3''{\cdot}0$ and 1396 stars instead of 1440 between $0''{\cdot}0$ and $1''{\cdot}0$.

13. The distribution of cross proper motions being given by the formula $\frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-1\cdot5}$, where $a = 1''{\cdot}4$, the proportion of stars between distances r and $r + dr$

$$= 2k^2 r e^{-k^2r^2} dr .$$

and the proportion of stars with linear proper motions between v and $v + dv$ (in the direction perpendicular to the solar motion)

$$= \frac{h}{\sqrt{\pi}} e^{-h^2v^2} . dv .$$

and $$k = 1''{\cdot}4\ h .$$

The value of k may be determined by equating the mean parallax determined from the above formula with that given by the parallactic motion of the stars. Denoting the mean parallax by $\bar{\pi}$

$$\bar{\pi} = \int_0^\infty 2k^2 e^{-k^2r^2} . dr ,$$

$$= k\sqrt{\pi} .$$

Similarly if the formula $0{\cdot}937 . \frac{d\tau}{a}\left(1+\frac{\tau^2}{a^2}\right)^{-1\cdot4}$, where $a = 1''{\cdot}2$, had been taken as representing the cross proper motions, the distance distribution would have been

$$1{\cdot}872\ k^{1\cdot8} r^{-0\cdot8} e^{-k^2r^2} . dr ,$$

and to determine k

$$1{\cdot}872\ k = \bar{\pi} .$$

14. If V be the Sun's linear velocity, and the apex be taken at declination $+35°$; then V cos $35° \bar{\pi}$ = Parallactic Motion.

The Carrington stars from the pole to 84° give the value $1''{\cdot}41$ for this quantity. Using all the stars I have since found the value $1''{\cdot}44$, but have kept the value $1''{\cdot}41$ in the following comparison

of the results given by the index 1·5 or 1·4 in the above formulæ. Taking the unit of distance R* to be that corresponding to a parallax of 1″·0, and adopting Campbell's value 19·5 km/sec for the Sun's velocity, $\frac{V}{R} = 412''$ a century, and $\overline{\pi} = \cdot 00418$.

15. With this value of π, the values of k for the two formulæ are ·00236 and ·00205 respectively, and the distribution in space is given as

$$2(\cdot 00236)^2 . r . e^{-(\cdot 00236)^2 r^2} . dr, \quad . \quad . \quad . \quad (6)$$

or

$$1 \cdot 872(\cdot 00203)^{1 \cdot 8} . r^{0 \cdot 8} . e^{-(\cdot 00203)^2 r^2} . dr. \quad . \quad . \quad (7)$$

The percentage of the Carrington stars at different distances given by these formulæ are shown in the following table:—

Distance.	Parallax.	Percentage of Stars. (1)	(2)
0 – 40	>″·025	0·9	1·0
40 – 100	″·025 to ″·010	5·0	4·7
100 – 200	·010 ·005	15·1	13·2
200 – 400	·005 ·0025	40·1	35·2
400 – 667	·0025 ·0015	31·5	32·9
667 – 1000	·0015 ·0010	7·1	11·9
>1000	<″·001	0·3	1·1

The distribution is also shown in the accompanying diagram (fig. 2). The most important feature is the comparatively close limits of parallax within which most of the stars lie, 70 per cent. having parallaxes between ″·005 and ″·0015.

Judging from the agreement with the proper motions, it would seem that the first of these formulæ holds best for the nearer and the second for the more distant stars, but the differences between the two are not large. In view of the fact that a small diminution of the assumed probable error of the proper motions gives a perfect accordance between the formula $\frac{d\tau}{a}\left(1 + \frac{\tau^2}{a^2}\right)^{-1 \cdot 5}$ and the observations, it appears to me that the distribution in distance of the stars in Carrington's Catalogue, and presumably of stars in other parts of the sky down to the limit of magnitude of the *Bonn Durchmusterung*, may be represented by a simple formula of the form

$$2k^2 r e^{-k^2 r^2} . dr.$$

Although the formula should not be pressed too far, it may be noticed that for small values of r the number of stars nearer

* There is need for a name for this unit of distance. Mr. Charlier has suggested Siriometer, but if the violence to the Greek language can be overlooked, the word *Astron* might be adopted. Professor Turner suggests *Parsec*, which may be taken as an abbreviated form of "a distance corresponding to a parallax of one second."

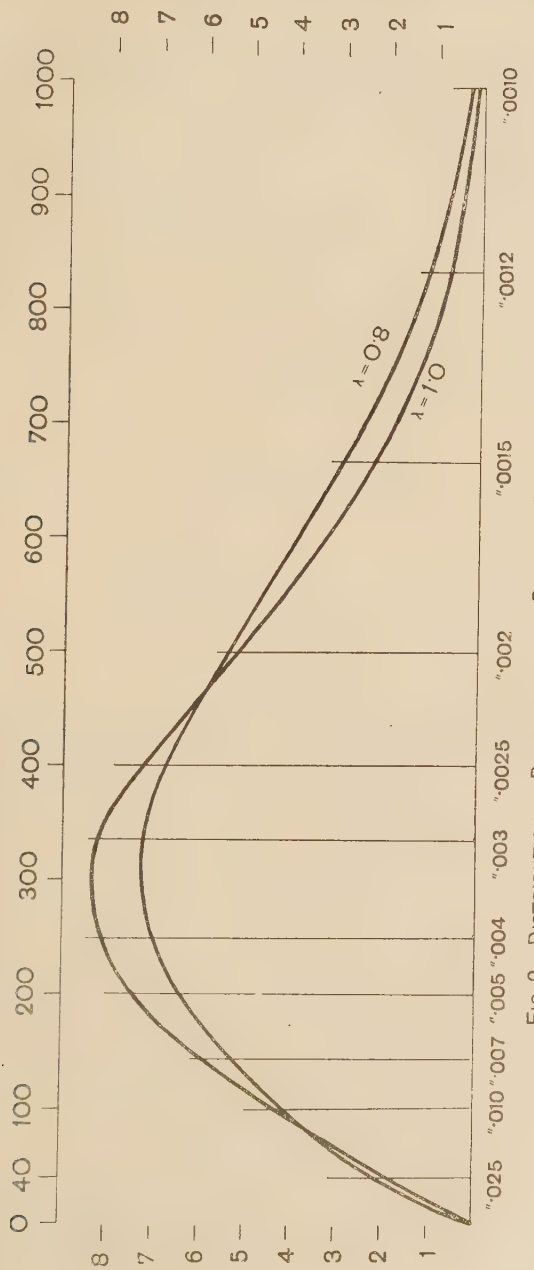


FIG. 2.—DISTRIBUTION IN DISTANCE OF THE STARS IN CARRINGTON'S CATALOGUE.

(1) When number between r and $r+dr \propto re^{-kr^2}$.
(2) " " $\rho^{0\cdot8}e^{-kr^2}$.

than r is proportional to r^2. Using the values given above, there are two stars in Carrington's Catalogue with parallax $> 0''{\cdot}1$. As the area covered is 250 square degrees, there are in the whole sky, at this rate, 320 stars within the B.D. limit of magnitude, with parallax greater than $0''{\cdot}1$.

16. I have examined the law $k^2 r\, e^{-kr}\, .\, dr$, but find that it does not give results at all in accordance with the observed proper motions.

17. The velocities of the stars are determined by h, and $h = k/a$. The values of h given by the two formulæ are sensibly the same; in both cases $h = {\cdot}00168$. The mean velocity $= \frac{1}{h\sqrt{\pi}} =$ $3''{\cdot}37$ per century as seen at a distance R. The mean linear velocity is therefore 16 km/sec.

18. The mean parallax of stars with given cross motion is readily found to be

$$\frac{\Gamma\left(\frac{\lambda}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{\lambda}{2}+1\right)} \cdot \sqrt{k^2 + h^2\tau^2}\,.$$

For $\lambda = 1$ this $= 1{\cdot}13\,({\cdot}00236^2 + {\cdot}00168^2\tau^2)^{\frac{1}{2}}$,

and for $A = 0{\cdot}8$ $= 1{\cdot}20\,({\cdot}00207^2 + {\cdot}00168^2\tau^2)^{\frac{1}{2}}$.

For large values of τ, these become ${\cdot}0019\ \tau$ and ${\cdot}0020\ \tau$ respectively, so that for $\tau = 20''$, the mean parallax is given as $''{\cdot}0038$ or $''{\cdot}0040$.

19. At Mr. Eddington's suggestion I have made a determination of the distribution of absolute luminosities, that is the number of stars of given luminosity as compared with the Sun in Unit volume of space. The assumption is made that the actual density and luminosity distribution is the same in all parts of the space occupied by the Carrington stars.

If the Sun's stellar magnitude be taken as $-26{\cdot}1$, then if removed to unit distance (parallax $1''$) it would appear as a star of $0^{\mathrm{m}}{\cdot}5$. Assuming that Carrington's limiting magnitude (the same as that of the *Bonn Durchmusterung*) is $10{\cdot}3$, then a star of luminosity $\frac{\odot}{8320}$ at unit distance would be just bright enough for inclusion in the Catalogue, as its apparent magnitude would be $\frac{1}{{\cdot}4} \times \log 8320$ or $9^{\mathrm{m}}{\cdot}8$ fainter than the Sun at this distance. Stars between the distances r and $r + dr$ must be of luminosity $> \frac{\odot}{8320}\, r^2$ to be included in the Catalogue.

Carrington's Catalogue may be taken as covering $\frac{1}{160}$ of the sky.

The number of stars in the volume $\frac{4\pi}{160} r^2 dr$ found in the Catalogue is

$$3735 \times 2 \times (\cdot 00236)^2 \,.\, re^{-(\cdot 00236r)^2} .\, dr \text{ by formula (6),}$$

or

$$3735 \times 1\cdot 872 \times (\cdot 00203)^{1\cdot 8} r^{0\cdot 8} e^{-(\cdot 00203r)^2}\, dr \text{ by formula (7).}$$

The number in unit volume $\frac{4\pi}{3} \times 10^6$, *i.e.* in the sphere whose radius corresponds to a parallax $''\cdot 01$ is obtained by multiplying the above expressions by $\frac{4\pi}{3}\ 10^6 / \frac{4\pi}{160}\ r^2\, dr \,.= \frac{160}{3}\ 10^6\ r^2 \,.\, dr$.

These stars are of luminosity $> \frac{\odot}{8320} r^2$. Calling this luminosity $x\odot$, $r^2 = 8320\, x$, and the following formula is found for N(x), the number of stars of luminosity greater than x times that of the Sun.

$$\mathrm{N}(x) = 24{,}400\ x^{-\cdot 5} \,.\, 10^{-\cdot 0201x}$$

or

$$23{,}600\ x^{-\cdot 6} \,.\, 10^{-\cdot 0149x}.$$

	Number of Stars.	
Luminosity greater than 200 times the Sun.	0	0
,, ,, 100 ,, ,,	24	48
,, ,, 50 ,, ,,	340	406
,, ,, 25 ,, ,,	1,530	1,450
,, ,, 10 ,, ,,	4,840	4,200
,, ,, 1 ,, ,,	23,200	22,800
,, ,, $\frac{1}{10}$th of the Sun	93,300	76,600

In connection with the luminosity of the stars, attention may be drawn to the fact that more than 95 per cent. of the stars in Carrington's Catalogue, and presumably of all the stars in the *Bonn Durchmusterung*, are brighter than the Sun. This percentage is the proportion of stars whose parallaxes are less than $0''\cdot 01$, and for a star at this distance to have an apparent magnitude of $10^{m}\cdot 3$, its luminosity must be greater than $\frac{\odot}{8320} \times 10{,}000$, or $1\cdot 2$ times that of the Sun.

20. The changes introduced by the adoption of the value of a $1''\cdot 44$ instead of $1''40$ and of the parallactic motion from $1''\cdot 41$ to $1''\cdot 44$ are very small. The velocities are practically unaltered, the mean being 16·0 km/sec. The mean parallax is increased from $''\cdot 00412$ to $''\cdot 00421$, and the value of k is increased from $''\cdot 00236$ to $''\cdot 00241$. There is of necessity some uncertainty in the determination of the parallactic motion of the Carrington stars, and further investigation may lead to an alteration of the value $1''\cdot 44$.

21. The proper motions on which this paper is based were determined under Mr. Hollis's direction in connection with the Astrographic work at the Royal Observatory. I am also indebted to him for checking other parts of the arithmetical work involved in dealing with the material.

The Distribution in Space of the Bright Stars.

By A. S. Eddington, M.A., M.Sc.

The problem considered is to find how the stars, which appear to us brighter than magnitude 5·8, are actually distributed as regards distance from the Sun. Stars of spectral types A and K have been examined separately, and in each case results have been obtained for two regions, one typical of high galactic latitudes and the other of low.

The results depend solely on the observed proper motions. No use is made of measured parallaxes or magnitude counts of stars. Radial motions have only been referred to (1) to justify the second assumption (*infra*), (2) for the adopted speed of the solar motion, viz., 19·5 km. per sec.

The results are given in Table V. As far as can be judged, the method has been successful in the case of the K stars. The results for the A stars are less satisfactory.

1. *The Assumptions.*

It is assumed that for stars of a definite spectral class—

(1) The distribution of the linear motions* follows the same law at all distances from the Sun.

(2) The components of the linear motions perpendicular to the line joining the Vertices (viz. the components unaffected by the phenomenon of star-streaming) are distributed according to the law of errors. The error distribution is ranged about a mean value, generally different from zero, because there is a component of the solar motion in this direction.

The assumption (1) is almost invariably made in investigations of this kind. It is not quite so innocuous as it appears, because, since the intrinsically fainter stars are eliminated by increasing distance, it really involves the assumption that the motions do not depend on the intrinsic luminosity of the stars. However, as we confine ourselves to one definite spectral class, the most obvious objection to it disappears.

The second assumption may, at least for stars of type A, be regarded as established by observation. I have taken the radial motions for type A given by W. W. Campbell (*L.O.B.*, No. 211) and compared them with the error-law. Omitting 40 stars near the Vertices (since we are concerned with components at right angles to the line joining the Vertices) we obtain the following comparison :—

Velocities.	Error Law.	Observed.
0– 4·95 km. sec.	53·4	55
4·95– 9·95	46·2	47
9·95–15·95	38·3	30
15·95–25·5	27·4	30
25·5 –40	6·7	10
>40	0	0

* *I.e.* of stars brighter than the limiting magnitude.

The error law is followed remarkably closely, the differences being apparently accidental.

It is important to note that these assumptions are made for each spectral class separately. From what we know of the differences in the motions of the A and K stars, there would be a danger in applying the assumptions to a heterogeneous mixture of the two types.

2. *Theory.*

Consider the stars in an area of the sky sufficiently limited to be treated as plane.

Let the proper motions be resolved into components along the direction towards the Vertex and at right angles to it (in the plane of the sky). Denote the latter components by η.

Let $-V$ = component of the solar motion (in linear measure) in the η direction.

Let $h(\eta)d\eta$ = Number of stars, whose component η of proper motion lies between η and $\eta + d\eta$.

$g(v)dv$ = Proportion of stars whose corresponding component of linear motion lies between v and $v + dv$.

$f(r)dr$ = Number of stars (in the region) at a distance r to $r + dr$; so that the density of the stars brighter than the limiting magnitude at a distance r from the Sun is proportional to $r^{-2}f(r)$.

Let further

$H(\eta) = \int_0^{\eta} h(\eta)d\eta$ = Number of stars whose component proper motion is between 0 and η.

$F(r) = \int_0^{r} f(r)dr$ = Number of stars whose distance is less than r.

The units of r, v, and η are made to correspond, so that

$$v = r\eta.$$

In accordance with assumption (2) let

$$g(v) = \frac{h}{\sqrt{\pi}} e^{-h^2(v-V)^2} \quad . \quad . \quad . \quad . \quad (1)$$

Since $v - V$ is the component of linear motion freed from the parallactic motion V.

We have the well-known fundamental equation

$$h(\eta)d\eta = \int_0^{\infty} f(r)dr \,.\, g(r\eta)rd\eta \quad . \quad . \quad . \quad (2)$$

expressing that the number of proper motions η to $\eta + d\eta$ is made up by multiplying the number of stars at a distance r by the fraction having linear motions between $r\eta$ and $r(\eta + d\eta)$, for successive steps of r.

Integrating by parts

$$h(\eta) = \Big[F(r) \,.\, rg(r\eta) \Big]_0^{\infty} - \int_0^{\infty} F(r) \frac{\partial}{\partial r} \big(rg(r\eta) \big) dr.$$

The first term vanishes at both limits, provided η is not zero, and moreover

$$\frac{\partial}{\partial r}\big(rg(r\eta)\big) \equiv \frac{\partial}{\partial \eta}\big(\eta g(r\eta)\big).$$

Thus $$h(\eta) = -\int_0^\infty \mathrm{F}(r)\frac{\partial}{\partial \eta}\big(\eta g(r\eta)\big)dr \quad . \quad . \quad (\eta \gtrless 0)$$

Therefore $$\int h(\eta)d\eta = -\int_0^\infty \mathrm{F}(r)\eta g(r\eta)dr \quad . \quad . \quad . \quad (3)$$

There is a discontinuity at $\eta = 0$, at which point our equations do not apply; $\mathrm{Lim}_{\epsilon=0}\int_0^\infty \mathrm{F}(r)\epsilon g(r\epsilon)dr$ is finite, and takes different values according as ϵ approaches 0 from the positive or negative side.

Thus (3) gives

$$\mathrm{H}(\eta) = \left.\begin{matrix}\mathrm{K}_1\\ \mathrm{K}_2\end{matrix}\right\} - \int_0^\infty \mathrm{F}(r)\eta g(r\eta)dr \quad . \quad . \quad . \quad (4)$$

where the constants of integration K_1 and K_2 refer respectively to positive and negative values of η. We do not at present need to determine their values.*

We shall first consider a particular form of $f(r)$, which is found particularly useful in practice.

If $$f(r) = re^{-h^2k^2r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\mathrm{F}(r) = 1 - e^{-h^2k^2r^2} \qquad \text{(neglecting constant multipliers).}$$

Therefore, by (1), (4), and (5)

$$\mathrm{H}(\eta) = \mathrm{K}_1 - \frac{h}{\sqrt{\pi}}\int_0^\infty (1 - e^{-h^2k^2r^2})\eta e^{-h^2(\eta r - \mathrm{V})^2}dr\,;$$

also $$\mathrm{H}(-\eta) = \mathrm{K}_2 + \frac{h}{\sqrt{\pi}}\int_0^\infty (1 - e^{-h^2k^2r^2})\eta e^{-h^2(-\eta r - \mathrm{V})^2}dr$$

$$= \mathrm{K}_2 - \frac{h}{\sqrt{\pi}}\int_{-\infty}^0 (1 - e^{-h^2k^2r^2})\eta e^{-h^2(\eta r - \mathrm{V})^2}dr.$$

Therefore

$$\mathrm{H}(\eta) + \mathrm{H}(-\eta) = \mathrm{K}_1 + \mathrm{K}_2 - \frac{h\eta}{\sqrt{\pi}}\int_{-\infty}^\infty (1 - e^{-h^2k^2r^2})e^{-h^2(\eta r - \mathrm{V})^2}dr.$$

$$= \mathrm{K}_1 + \mathrm{K}_2 - 1 + \frac{h\eta}{\sqrt{\pi}}\int_{-\infty}^\infty \exp.-\{h^2r^2(k^2 + \eta^2) - 2h^2\eta r\mathrm{V} + h^2\mathrm{V}^2\}dr.$$

* Actually K_1 and K_2 are the whole number of positive and negative proper motions respectively. The formula (4) has the following physical meaning: the number of proper motions greater than η is found by multiplying the number of stars nearer than r by the proportion with linear velocities between ηr and $\eta(r + dr)$ for successive steps of r. It is, of course, possible to arrive at this proposition by general reasoning; but, notwithstanding its apparent simplicity, it is not very easy to grasp.

Putting $\eta = 0$, we see that $K_1 + K_2 - 1$ must vanish; and setting

$$\eta = nk$$
$$\tau = h\mathrm{V}$$

and evaluating the integral we find

$$\mathrm{H}(\eta) + \mathrm{H}(-\eta) = \frac{n}{(1+n^2)^{\frac{3}{2}}} e^{-\frac{\tau^2}{1+n^2}} \quad . \quad . \quad . \quad (6)$$

The right-hand side is denoted by $R_1(n)$. The quantity $\tau = h\mathrm{V}$ can be found from the ratio of the number of positive to the number of negative proper motions η, as will be shown when we come to deal with the observations. Its value is therefore known.

The result thus shows that, if we can find a value of k, such that the proportion of stars having η component motions numerically less than nk is represented by $R_1(n)$, then the distribution is given by $f(r) = re^{-h^2k^2r^2}$.

Since (knowing τ) we can tabulate $R_1(n)$, this comparison with observation is not difficult.

Evidently a distribution of the form

$$f(r) = a_1 re^{-h^2k_1^2r^2} + a_2 re^{-h^2k_2^2r^2}$$

can be treated in the same manner. This happens to be the form, which we shall find most useful.

General Case.—It seems tempting to assume that the distribution can be expanded in the form

$$f(r) = (a_1 r + a_3 r^3 + a_5 r^5 + \ldots\ldots) e^{-h^2k^2r^2}$$

The resulting distribution of proper motions is

$$\mathrm{H}(\eta) + \mathrm{H}(-\eta) = \mathrm{N}_1\mathrm{R}_1(n) + \mathrm{N}_2\mathrm{R}_2(n) + \mathrm{N}_3\mathrm{R}_3(n) + \ldots\ldots$$

where N_1, N_2, N_3 are the total number of stars corresponding to the successive terms, and it may be shown that

$$\left.\begin{aligned}
\mathrm{R}_1(n) &= \frac{n}{(1+n^2)^{\frac{3}{2}}} e^{-\frac{\tau^2}{1+n^2}} \\
\mathrm{R}_2(n) &= \left\{ \frac{n^3\tau^2}{(1+n^2)^{\frac{5}{2}}} + \frac{\frac{3}{2}n + n^3}{(1+n^2)^{\frac{3}{2}}} \right\} e^{-\frac{\tau^2}{1+n^2}} \\
\mathrm{R}_3(n) &= \left\{ \tfrac{1}{2} \frac{n^5\tau^4}{(1+n^2)^{\frac{9}{2}}} + \frac{(\frac{5}{2}n^3 + n^5)\tau^2}{(1+n^2)^{\frac{7}{2}}} + \frac{\frac{15}{8}n + \frac{5}{2}n^3 + n^5}{(1+n^2)^{\frac{5}{2}}} \right\} e^{-\frac{\tau^2}{1+n^2}}
\end{aligned}\right\} \quad (7)$$

It is found, however, that R_2 or R_3 can be replaced very closely (as far as practical applications are concerned) by an R_1 function with a different value of k. I have used R_2 (and in some cases R_3) freely in trying to represent the observations, but have always found that the results are a little inferior to those that can be obtained with a combination of R_1's. The difference is generally very slight.

Another general expansion

$$f(r) = (a_0 + a_2 r^2 + a_4 r^4 + \ldots\ldots) e^{-h^2k^2r^2}$$

can be similarly treated by considering $\mathrm{H}(\eta) - \mathrm{H}(-\eta)$. I tried

this also before using the other series, but it seems to be less suited to the observations.

My conclusion, that the form of distribution law that best fits the observations is $f(r) = re^{-h^2k^2r^2}$, agrees with that of Dr. Dyson (p. 338). Using the form $r^\lambda e^{-h^2k^2r^2}$, where λ is not restricted to integral values, he found the best agreement was obtained by taking λ between 0·8 and 1·0.

3. *The Observations.*

Two regions of the sky have been examined: (1) the region R.A. $21^h\ 36^m$ to $2^h\ 24^m$, Dec. $-36°$ to $+36°$, together with the antipodal region; (2) the region from the North Pole to Dec. $+50°$, together with the antipodal region. For each of them the stars of Type A (*i.e.* A to A9) and K have been considered separately; the discussion has been confined to stars of $6^m{\cdot}0$ and brighter, for which Boss's Catalogue may be considered complete. As the magnitudes of the catalogue need a correction to reduce to the Harvard scale, the limiting magnitude in this discussion is approximately 5·8 (Harvard).

The centres of the two regions are at galactic latitudes, 62° and 28° respectively. We may thus consider them as fairly typical of high and low galactic latitudes. Distinguishing these by the letters H and L, we shall indicate the four groups of stars shortly as

HA, HK, LA, LK.

The H region is the more favourably placed for the purposes of this investigation; the L region was examined afterwards to serve as a check.

The two regions are of equal area.

Determination of τ.—Starting from an initial line pointing towards the Vertex R.A. 94° Dec. +12°, the number of motions in the four quadrants are:—

TABLE I.

Number of Proper Motions.

	HA.	HK.	LA.	LK.
First quadrant . . .	116	110	86½	73
Second ,, . . .	42	52	27½	47
Third ,, . . .	8	11	34	39½
Fourth ,, . . .	19	28	131	117½
η positive (quadrants 1 and 2)	158	162	114	120
η negative (quadrants 3 and 4)	27	39	165	157

Now the proportion of negative components to the whole number is

$$\int_{-\infty}^{0} e^{-h^2(v-V)^2}dv : \int_{-\infty}^{\infty} e^{-h^2(v-V)^2}dv,$$

which is equal to

$$\int_{-\infty}^{-hV} e^{-x^2}dx : \int_{-\infty}^{\infty} e^{-x^2}dx.$$

$$= \frac{1}{\sqrt{\pi}}\int_{hV}^{\infty} e^{-x^2}dx.$$

Thus by means of a table of the probability integral, we find

	HA	HK	LA	LK
$hV = \tau =$	·75	·61	−·16	−·12

It is satisfactory to note in Table I. that the ratio

$$\frac{\text{first quadrant}}{\text{fourth quadrant}} = \frac{\text{second quadrant}}{\text{third quadrant}}$$

within reasonable limits of error. This confirms our assumption that the η components are free from any effect of star-streaming.

The η Components of Motion.—The number of stars having η components within given limits is shown in Table II.

TABLE II.

Limits of $\pm\eta$.	Group HK.		Group LK.		Group HA.		Group LA.	
	η pos.	η neg.	η pos.	η neg.	η pos.	η neg.	η pos.	η neg.
0 – 9	13	15	40	35	29	19	42	52
10 – 19	25	6	19	35	34	3	33	45
20 – 29	21	5	16	29	25	4	17	29
30 – 39	14	2	10	17	19	0	5	17
40 – 49	13	1	7	11	14	0	6	11
50 – 59	14	1	3	6	6	0	2	3
60 – 69	14	2	4	3	4	0	3	5
70 – 79	8	2	6	8	5	0	1	2
80 – 89	5	0	3	3	2	0	3	0
90 – 99	0	1	3	1	3	1	0	0
100 – 150	15	1	5	4	11	0	0	1
150 – 200	10	1	3	3	4	0	2	0
200 – 500	6	1	1	2	2	0	0	0
>500	4	1	0	0	0	0	0	0
Totals	201		277		185		279	

(The unit of η is ″·001 per annum.)

Adding together the positive and negative components, we obtain $h(\eta) + h(-\eta)$. In the next table the results have been smoothed graphically, and a small correction applied to allow for the effect of the probable accidental errors of Boss's proper motions.*

* See p. 359. From the data given in the catalogue the average probable error of a component proper motion is between ″·005 and ″·006 per annum.

TABLE III.

No. of Stars within given limits of proper motion component η (irrespective of sign). Results smoothed, and a correction applied for accidental error of the determinations of proper motion.

$\pm\eta$.	HK.	LK.	HA.	LA.
0 – 9·5	28½	79	50⅓	98
9·5 – 19·5	27	55	37	78½
19·5 – 29·5	24	40½	26½	39½
29·5 – 39·5	21	26½	18½	23
39·5 – 49·5	18	17½	11	15
49·5 – 59·5	14	12½	8	9½
59·5 – 69·5	10½	10	6	5½
69·5 – 79·5	8	8	4½	3
79·5 – 89·5	6	6	3½	2
89·5 – 100	5	4½	3	1
100 – 150	16	9	9	1½
150 – 200	11	6	4	1½
200 – 500	7	3	2	0
>500	5	0	0	0
Totals	201	277½	183½	278

(Unit of $\eta = ''\cdot001$ per annum.)

We have now to represent these by combinations of R_1, R_2, or R_3 functions. The values of τ for each group having been given above, we can draw graphs of the corresponding functions. The process of finding a suitable combination by trial and error is not difficult. I find the observations are best represented as follows:—

$$\text{If} \qquad H(\eta) + H(-\eta) = N_1 R_1\left(\frac{\eta}{k_1}\right) + N_2 R_1\left(\frac{\eta}{k_2}\right) \quad . \quad . \quad (8)$$

so that $$f(r) = 2N_1 h^2 k_1^2 r e^{-h^2 k_1^2 r^2} + 2N_2 h^2 k_2^2 r e^{-h^2 k_2^2 r^2} \quad . \quad . \quad (9)$$

the constants are

	HK	LK	HA	LA
N_1	124	237½	153½	310
k_1	37·7	30·6	19·4	25·1
N_2	77	40	30	−32
k_2	80·0	80·0	75·0	60·0

A comparison of the observed distribution with that calculated from these constants is given in Table IV.

Then by (9) the distribution law as regards distance can be found.

TABLE IV.

Comparison of adopted formulæ with observation.

η	HK. Calc.	HK. Obs.	LK. Calc.	LK. Obs.	HA. Calc.	HA. Obs.	LA. Calc.	LA. Obs.
< 9·5	27·6	28·5	74·3	79	45·7	50·5	103·9	98
< 19·5	55·3	55·5	135·4	134	86·9	87·5	176·7	176·5
< 29·5	80·7	79·5	176·7	174·5	115·0	114	219·8	216
< 49·5	117·2	118·5	221·5	218·5	144·2	143·5	255·6	254
< 69·5	140·7	143	242·2	241	156·2	157·5	268·3	269
>100	37·6	39	19·3	18	15·7	15	5·1	3
>150	...	...	10·0	9	8·4	6	2·4	1·5
>200	12·3	12	...	...	...	...	...	...

Evaluation of h.—To convert the distances into known measure it is necessary to determine the value of h, which is inversely proportional to the mean peculiar motion (η component) of the stars. It is not improbable that this may be different in high and low galactic latitudes, and it certainly differs for the A and K stars.

A simple method of finding h is as follows:—

$$\tau = h\mathrm{V}.$$

But τ has been determined for each region; V is the component of the solar motion in the η direction and is known in linear measure. Thus h can be found. Unfortunately, however, V and τ are very small for LK and LA; so that for them the equation is indeterminate.

A better method is to calculate the mean parallactic motion for each group, and hence, by the known value of the solar motion, determine the mean parallax $\overline{\varpi}$. From (9) it can be shown that

$$\overline{\varpi} = \frac{\sqrt{\pi}(\mathrm{N}_1 k_1 + \mathrm{N}_2 k_2)h}{\mathrm{N}_1 + \mathrm{N}_2} \quad . \quad . \quad . \quad (10)$$

which determines h.

In calculating $\overline{\varpi}$ it is always necessary to reject a few very large proper motions; but it is clear that, if we do so, there must be some compensation on the right-hand side of (10). I have generally rejected about 3 per cent., taking care to reject about the same proportion of motions in the different directions. These will be mostly, though not absolutely, the nearest stars. We shall not be far wrong in assuming that they are the nearest stars. Then, when r is small,

Number of stars between r and $r+dr$ is

$$2\mathrm{N}_1 h^2 k_1{}^2 r + 2\mathrm{N}_2 h^2 k_2{}^2 r \qquad \text{approximately,}$$

Therefore number of stars nearer than R is

$$(\mathrm{N}_1 k_1{}^2 + \mathrm{N}_2 k_2{}^2) h^2 \mathrm{R}^2,$$

and sum of parallaxes is

$$2(N_1k_1^2+N_2k_2^2)h^2R.$$

Thus the mean parallax of these stars is $\frac{2}{R}$.

And the mean parallax of the remainder rejecting the p nearest stars is

$$\bar{\varpi}=\frac{\sqrt{\pi}(N_1k_1+N_2k_2)-\frac{2p}{hR}}{N_1+N_2-p}\cdot h \quad . \quad . \quad . \quad (11)$$

where hR is given by $(N_1k_1^2+N_2k_2^2)h^2R^2=p$.

For the four groups the mean parallactic motions (corrected for foreshortening, by multiplying by the cosecant of the angular distance of the region from the apex *) are :—

	HK	LK	HA	LA
Parallactic motion	55·0	40·5	41·1	43·4
No. of stars rejected	7	7	6	8

and the mean parallaxes calculated by (11) are

$$\bar{\varpi}= 76{\cdot}6h \qquad 54{\cdot}9h \qquad 39{\cdot}0h \qquad 32{\cdot}6h$$

The question arises, whether the solar motion has the same value for both types; according to Campbell's latest determinations it is about 4 km. per sec. less for Type A. In view, however, of the uncertainties which still exist, we shall adopt the same value 19·5 km./sec. for all.

With this value, a parallactic motion of one unit corresponds to a parallax of 0″·000243. If we take as unit of r a distance corresponding to a parallax of 1″, the values of h are found to be

HK	LK	HA	LA
·000174	·000179	·000257	·000323

Since these values are of great importance in our investigation, and the manner in which they are derived may seem rather complicated, it will be of interest to put them into another form, which shows their physical meaning more plainly.

The mean residual velocity resolved in any direction (after eliminating the solar motion and star-stream motions) is equal to $1/h\sqrt{\pi}$. By converting the above values of h into suitable units, we find that this residual velocity takes the values

HK	LK	HA	LA
15·3	14·9	10·4	8·3 km. per sec.

These values compare well with the generally accepted mean radial velocities of the respective types (which, however, are generally given uncorrected for drift motion and therefore appear some-

* This is taken at R.A. 270° Dec +30° for A stars.
and R.A. 270° Dec +40° for K stars.
in accordance with L. Boss's determinations, *A.J.*, Nos. 623–4.

what larger). The difference between HA and LA, if real, would be accounted for by a general tendency to move parallel to the galactic plane, a phenomenon which has sometimes been suspected; it should be remembered in making comparisons that these are transverse components of motion, and a transverse velocity in low galactic latitudes is properly compared with a radial component in high galactic latitudes, and *vice versa*.

We have now all the numerical values necessary to insert in equation (9), or more conveniently in its integrated form :—

$$F(r) = N_1(1 - e^{-h^2k_1^2r^2}) + N_2(1 - e^{-h^2k_2^2r^2}).$$

The following distribution of stars is derived :—

TABLE V.

Distribution of Stars brighter than $5^m{\cdot}8$.

Limits of Parallax.	Limits of Distance.	Type K.		Type A.		Type A. Low Lat.
		High Lat.	Low Lat.	High Lat.	Low Lat.	Northern Half.*
"						
...	0	11·7	9·2	7·9	5·7	7·5
·04	25	30·7	24·1	18·2	20·9	21·4
·02	50	35·3	31·5	18·8	39·5	32·7
·0133	75	31·4	31·8	17·6	50·3	38·5
·01	100	24·8	31·0	16·7	48·8	40·9
·008	125	19·0	28·7	16·3	40·4	39·0
·0067	150	14·5	26·5	16·0	29·5	34·5
·0057	175	11·3	23·3	15·0	19·6	27·9
·005	200	8·2	19·5	13·2	11·5	21·3
·0044	225	5·7	15·6	11·2	6·3	15·3
·004	250					
<·004	>250	8·4	36·3	32·6	5·5	25·0

The figures in the last five columns give the number of stars between the distances in the second column.

Each region occupies ·235 of the whole sky, and the figures give the actual numbers of stars (*i.e.* they are absolute and not merely proportional).

The unit of distance corresponds to a parallax of 1″.

4. *Discussion of the Results.*

It is not easy to form an idea of how reliable Table V. is likely to be. The table is offered as the result of one method of attack-

* The reason for giving this column is explained later, p. 358.

ing the problem; a final solution will only be reached when two or more different lines of reasoning converge to the same numerical results.

Hitherto our knowledge of this question has been very scanty, so that a result within 25 per cent. of the truth may be regarded as a useful approximation. I think that for the K stars the numbers are probably good to this extent; as regards Type A, I fear that the results are doubtful.

In a former paper (*M.N.*, lxxii. p. 386, Table VII.) I gave a provisional table of the distribution of the Boss stars in distance. The region there treated was the high region of the present paper, but the results are not strictly comparable, as the stars fainter than $5^{m}\cdot8$ and also the types F, G, and M were then included. The present numbers, however, appear to confirm the main features of the provisional table satisfactorily. Thus comparing it with the present numbers for HA and HK combined:—

Parallax.	Provisional Table.	HK and HA.
$>''\cdot04$	5·4 per cent.	5·1 per cent.
$<''\cdot004$	12·4 ,,	10·7 ,,
·02 to ·008	37·8 ,,	37·6 ,,

This confirmation rather strengthens the results for the distribution of linear stellar motions which were obtained in that paper.

The comparatively small range of parallax, which includes the great majority of the stars, is a noticeable feature. About 75 per cent. are included between parallaxes $''\cdot02$ and $''\cdot004$. Professor Kapteyn's numbers (*A.J.*, 566) correspond to a distribution considerably more spread out.

Type K.

The numbers appear to accord well with the usual views as to stellar distribution. It is supposed that near the Sun the actual density of the stars is fairly uniform; and that in the plane of the galaxy this uniform distribution extends a great distance, whilst in directions perpendicular to the plane the density begins to thin out rather rapidly. From the figures in the table it appears that there is little difference in the two directions until a parallax of $0''\cdot01$ is reached; after that, the decrease in density in the high latitude becomes more and more noticeable, so that at a parallax $0''\cdot004$ it is not more than a third of the density in the galactic plane.

It seems likely that the unit of r in the Low Region is rather smaller than (about $\frac{7}{8}$) that of the High Region (a difference due to the accidental error of the determination of h). If this were corrected, the first three numbers in the two columns would agree closely, and for the larger distances the excess of stars in the Low Region would be rather increased.

If we suppose that as far as a parallax 0″·004 there is no appreciable falling off of density in the galactic plane, we can find the absolute luminosity curve of the K stars. For our table and formulæ give the number of stars brighter than a limiting luminosity, which depends on r. This limiting luminosity is

$$L = \left(\frac{r}{11{\cdot}5}\right)^2 \quad . \quad . \quad . \quad . \quad (12)$$

where the luminosity L is expressed in terms of the Sun as unit, and the Sun's stellar magnitude is taken to be $-26{\cdot}1$.

Thus the density $f(r)/r^2$ is the density of stars brighter than L, and by varying r we vary L, and thus trace out the luminosity curve.

We find that the number of stars brighter than L in a sphere of radius 100 units (parallax = ″·01) is equal to

$$\frac{10^6}{3 \times {\cdot}235} \cdot \frac{f(r)}{r^2},$$

and inserting the value of $f(r)$ for region LK, this becomes

$$\frac{1}{r}\{20210\, e^{-{\cdot}000030\, r^2} + 23260\, e^{-{\cdot}000205\, r^2}\} \quad . \quad . \quad (13)$$

From (12) and (13) the following table has been calculated:—

TABLE VI.

Stars of Type K.

(Number in a volume of $4{\cdot}2 \times 10^6$ units of space,—equal to a sphere whose radius corresponds to a parallax 0″·01.)

Luminosity (Sun = 1).	Number of Stars.
> 500	10·8
400 to 500	7·3
300 ,, 400	12·9
200 ,, 300	6·3
150 ,, 200	25·0
100 ,, 150	51·9
50 ,, 100	139·0
25 ,, 50	251·9
10 ,, 25	500
1 ,, 10	2700
Total brighter than the Sun	3725

We cannot extend the table to stars having a luminosity less than the Sun, for such stars must be within 11 units of the Sun to appear of magnitude 5·8. There are only three LK stars within this radius.

Type A.

The figures for the A stars do not fall in with our preconceived ideas. In particular, at very large distances, we find fewer A stars in the galactic plane than at right angles to it—a result which it is hardly possible to accept. The results for the High Region seem fairly satisfactory, showing a greater spreading of these stars in distance than was the case in Type K; but failing other confirmation, it is somewhat arbitrary to prefer them to the figures of the Low Region.

The discrepancy is not easily removed. We have

in HA 185 stars with a mean parallactic motion of 41·1,

and in LA 279 „ „ „ „ 43·4.

We can see without further analysis that it is impossible to regard the extra ninety stars in the Low Region as being added in the distant parts of the distribution. It is thus impossible to have the same kind of relation between the High and Low Regions that we found for the Type K stars.

The unexpectedly large parallactic motion of the A type stars in low galactic latitudes was noted by Boss (*A.J.*, 623–4), who used a much greater number of stars than we have done. He found—

Types B_8—A_4.

Galactic Latitude.	Parallactic Motion.	Number of Stars.	Relative Density.
0° to ±10°	38·4	422	2·00
±10° „ ±30°	41·2	620	1·57
±30° „ ±90°	42·0	605	1

Thus, although we see twice as many of these stars in low galactic latitudes as in high, their mean distance (indicated by the parallactic motion) is hardly appreciably different. One hesitates to discard the very simple view that the increased number in the galactic plane is due to the existence of more distant stars in this direction; but it must be admitted that the evidence of the parallactic motions is unfavourable to it, so far as the A stars are concerned.

Having some reason to suspect that the proper motions near the South Pole were affected by some error or local drift, I made a separate solution for the northern half of LA. The figures (doubled in order to be comparable with those for the full areas) are given in the last column of Table V. These figures are in some respects more acceptable than those for the whole region. The number of very distant stars tends to practically the same limit in high and low latitudes, instead of being greater in the high latitudes. But the puzzling feature, that the excess of stars in the low region consists of a belt of stars near the mean distance, remains unaffected.

On account of these uncertainties I do not give any investigation of the luminosities of the A stars; but it is clear that the results would closely resemble those for the K stars. This can be seen by comparing the last column of Table V. with the column for Type K, Low Latitude.

On a Formula for Correcting Statistics for the Effects of a known Probable Error of Observation. By A. S. Eddington, M.A., M.Sc.

In astronomical investigations it often happens that we construct a table to exhibit the number of stars having successive values of a certain measured property, *e.g.* the number of stars of successive magnitudes, or the number between successive limits of proper motion. In general the observations are subject to a probable error, which is at least approximately known. The error is in general small, otherwise our table would not be of much value, but it must have some effect on the numbers of the table. It is not, however, usual to take account of the probable error, probably because it is not generally realised that it can be eliminated and an improved table formed in a very simple way.

To avoid the awkwardness of general terms, suppose we are concerned with counts of stars between given limits of magnitude, and, knowing the average accidental error of our determinations of magnitude, we wish to apply corrections to our counts to eliminate these errors.

Let $u(m)dm$ = observed number of stars between magnitudes m and $m+dm$
$v(m)dm$ = true number.

Let the probable error of the observed magnitudes be $0{\cdot}477/h$, so that the frequency of an error x is proportional to $e^{-h^2x^2}$.

We have $$u(m) = \frac{h}{\sqrt{\pi}} \int_{-\infty}^{\infty} v(m+x) e^{-h^2x^2} dx \,;$$

for, of the stars having a true magnitude $m+x$, the proportion $\frac{h}{\sqrt{\pi}} e^{-h^2x^2}$ will have an error of measurement $-x$, and will therefore be observed as of magnitude m.

By the symbolic form of Taylor's theorem

$$v(m+x) = e^{x\frac{d}{dm}} \cdot v(m),$$

therefore $$u(m) = \frac{h}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{x\frac{d}{dm} - x^2h^2} \cdot v(m) dx \,.$$

The integral is of the well-known form

$$\int_{-\infty}^{\infty} e^{-a_1x - a_2x^2} dx = \sqrt{\frac{\pi}{a_2}} exp \frac{a_1^2}{4a_2},$$

thus integrating

$$u(m) = exp\left(\frac{1}{4h^2} \frac{d^2}{dm^2}\right) \cdot v(m) \,;$$

therefore $$v(m) = exp.-\left(\frac{1}{4h^2}\frac{d^2}{dm^2}\right).u(m)$$
$$= u(m) - \frac{1}{4h^2}u''(m) + \frac{1}{2!}\left(\frac{1}{4h^2}\right)^2 u^{iv}(m) - \ldots$$

The result so far is accurate.

For a small accidental error, confining ourselves to the first two terms, the result can be given in a form convenient for computation as follows:—

The tabular second difference for intervals a
$$= u(m+a) + u(m-a) - 2u(m) = a^2u''(m) \text{ approximately.}$$

Also $$\frac{1}{2h} = 1{\cdot}046 \times \text{probable error.}$$

Thus the approximate correction is
$$-\left(\frac{1{\cdot}046 \times \text{probable error}}{\text{tabular interval}}\right)^2 \times \text{tabular second difference.}$$

Note on the Convergency of the Series.—The Astronomer Royal has pointed out to me that the series in some typical cases is divergent, *e.g.* $u(m) = (1+m^2)^{-1}$. The operator $\frac{d^{2n}}{dm^{2n}}$ introduces a factor of the order $2n!$, whilst the divisor is only $n!$. Apparently, however, in these cases the expansion is asymptotic; and it seems certain that the first few terms give the approximate correction quite correctly. The divergence arises from using the Taylor expansion beyond its range of convergence.

The difficulty does not really arise in practice. In a table with a finite number of entries, we are actually dealing with a polynomial, in which case all the series terminate, and no question of divergence arises. Thus, if a is the tabular interval, the tabular values of $v(m)$ may be represented from $m - na$ to $m + na$ by a polynomial of the $2n^{\text{th}}$ degree, say $v_{2n}(m)$. By taking n sufficiently great the whole range of v, which contributes appreciably to $u(m)$, can evidently be included. Beyond the limits $m \pm na$, the divergence between $v_{2n}(m)$ and $v(m)$ will generally increase rapidly; it can be shown, however, that
$$\int_{na}^{\infty} x^{2n} e^{-h^2x^2} dx$$
tends to zero as n is increased, and hence the part of $v_{2n}(m)$ beyond the limits $m \pm na$ will (when n is great) make no contribution to $u(m)$.

Thus the polynomial and the true function agree for the $2n+1$ tabular entries $m - na$ to $m + na$, and beyond these limits they both make contributions to $u(m)$, which tend to zero. Hence the polynomial can be used for our purpose, and a terminating series results.

Substituting the polynomial only requires that we should deduce the differential coefficients from the tabular differences—a procedure which we should naturally adopt in any case.

Note on the Possibility of Refraction by the Solar Atmosphere. Papers of the I.U.S.R. Computing Bureau. No. VIII. By R. S. Capon.

(*Communicated by Prof. H. H. Turner.*)

In discussions of sun-spot movements, it appears to be always assumed that any refraction caused by the sun's atmosphere is negligible, but no direct attempt seems to have been made to discover whether this is so or not. The following brief analysis suggests that the motions of the spots are largely influenced by their position on the sun's disc with respect to the earth, and the hypothesis of refraction offers at all events a qualitative explanation of the effects.

Owing to the smaller magnitude of the irregular movements in latitude of the spots, the latitude drift appeared to offer greater facilities for a preliminary inquiry. The chief difficulty encountered was the elimination of the effect due to possible non-coincidence of the true and adopted axes of rotation of the sun. The following method was employed:—From the Greenwich volumes the heliographic latitudes of spots at longitude 60° east and west of the central meridian were tabulated. (For the latitude at any meridian, the mean of the latitudes at longitudes within 10° on either side of it was taken.) The mean of the two values was then compared with the latitude of the spot as it crossed the central meridian. If λ_l denotes the latitude at longitude l with respect to the central meridian, the expression

$$\tfrac{1}{2}(\lambda_{+60}+\lambda_{-60})-\lambda_0$$

denotes a movement of the spot towards the equator if its sign is positive, and from the equator if the sign is negative. (The numerical values of south latitudes were taken, so that the signs are the same for deflections towards the equator in both hemispheres.) The mean values of $\frac{1}{2}(\lambda_{+60}+\lambda_{-60})-\lambda_0$ for all spots crossing the central meridian in February, March, and April, and in August, September, and October of the years 1874–1910, are exhibited in Table I. The unit is 1°·0. The suffix denotes the number of spots in the mean.

TABLE I.

$\frac{1}{2}(\lambda_{60}+\lambda_{-60})-\lambda_0$.

Month Group. \ Hel. Lat.	+25	+20	+15	+10	0	−10	−15	−20	−25
Feb. Mar. Apr.	$-0^\circ.27_3$	$-0^\circ.19_{22}$	$+0^\circ.05_{35}$	$+0^\circ.02_{52}$	$+0^\circ.07_{49}$	$0^\circ.00_{65}$	$-0^\circ.05_{57}$	$+0^\circ.01_{52}$	$-0^\circ.01_{13}$	$-0^\circ.16_{10}$
Aug. Sept. Oct.	$-0^\circ.19_8$	$-0^\circ.22_{21}$	$-0^\circ.17_{29}$	$-0^\circ.14_{73}$	$+0^\circ.03_{56}$	$+0^\circ.03_{62}$	$-0^\circ.09_{22}$	$+0^\circ.01_{51}$	$+0^\circ.03_{26}$	$-0^\circ.28_4$

The latitude of the earth on the sun attains its maximum positive value at the middle of March, and its maximum negative value at the middle of September. Consequently, if the negative values of the outer groups are due to refraction, we should expect the positive and negative values to be symmetrically situated with respect to latitude $+6°{\cdot}0$ in the March group, and $-6°{\cdot}0$ in the September group, for these are approximately the mean heliographic latitudes of the earth for the groups. In Table II. the south latitudes of the September group are placed below the corresponding north latitudes of the March group, so that the earth may be considered to be at latitude $+6°{\cdot}0$ in both. A third row exhibits the weighted means of the two rows above it. The unit is $1°{\cdot}0$. E denotes the position of the earth in heliographic latitudes.

TABLE II.

Month Group. / Hel. Lat.	+25	+20	+15	+10 E	0	−10	−15	−20	−25
Feb. Mar. Apr.	$-0{\cdot}27_3$	$-0{\cdot}19_{22}$	$+0{\cdot}05_{35}$	$+0{\cdot}02_{52}$	$+0{\cdot}07_{49}$	$0{\cdot}00_{65}$	$-0{\cdot}05_{57}$	$+0{\cdot}01_{52}$	$-0{\cdot}01_{13}$	$-0{\cdot}16_{10}$
Aug. Sept. Oct.	$-0{\cdot}28_4$	$+0{\cdot}03_{26}$	$+0{\cdot}01_{51}$	$-0{\cdot}09_{22}$	$+0{\cdot}03_{62}$	$+0{\cdot}03_{56}$	$-0{\cdot}14_{73}$	$-0{\cdot}17_{29}$	$-0{\cdot}22_{21}$	$-0{\cdot}19_8$
Weighted Means	$-0{\cdot}28_7$	$-0\ 07_{48}$	$+0{\cdot}03_{86}$	$-0{\cdot}01_{74}$	$+0{\cdot}05_{111}$	$+0{\cdot}01_{121}$	$-0{\cdot}10_{130}$	$-0{\cdot}05_{81}$	$-0{\cdot}14_{34}$	$-0{\cdot}17_{18}$

Table II. indicates that as a spot crosses the sun's disc it moves towards the pole of the hemisphere in which it is situated, until it reaches the central meridian, when the motion is reversed, and for the remainder of the distance it moves away from the pole, and this effect becomes more marked as the latitude increases. Moreover, the greater number of positive values in each case in the hemisphere in which the earth is situated suggest that the effect is less marked in this hemisphere, though, in view of the irregularity of the numbers, not much stress is to be laid on the latter conclusion. Now, in fig. 1, if S′ is the position of a spot, P the pole of the sun, E the point where the line joining the centre of the sun to the earth cuts the surface of the sun, the spot would be displaced by refraction to S, where S S′ is a function of the angular distance S′E. The displacement in latitude is S S′ cos S′S P, and therefore, unless the law of refraction is such that S S′ cos S′S P is constant, there will be an effect similar to that shown in Table II. It should be noticed that an error in the adopted axis of rotation will not explain the result, for it would give rise to all positive values in one group and all negative values in the other. The only other solution would appear to be an actual *acceleration* in latitude of the spots towards the equator in each hemisphere, the acceleration increasing with the latitude, and a little greater in the south than in the north hemisphere during the months of February, March, and April, and greater in the north than in the south in the months of August, September, and October. Apart from the artificial nature of the alternative explanation, the hypothesis of refraction will be found to be supported by the results of the analysis of the motions in longitude.

This analysis was conducted in the following way:—The angular velocity was determined from the motions of spots at the central meridian, and at the meridians 65° and 75° east and west. The angular velocity was obtained thus:—The difference of two values of heliographic longitude was taken, one on each side of the meridian under investigation. In the case of the meridians 65° and 75° east and west, the values were within 10° of the meridian; in the case of the central meridian, within 20°. In each case the difference was divided by the time-interval between the observations, and the results tabulated under the head "residual angular velocities." The residual angular velocity is therefore the number of degrees per day by which the angular velocity of the spot exceeds or falls short of the angular velocity of the adopted zero of heliographic longitude (rotating at the rate of 14°·18 in

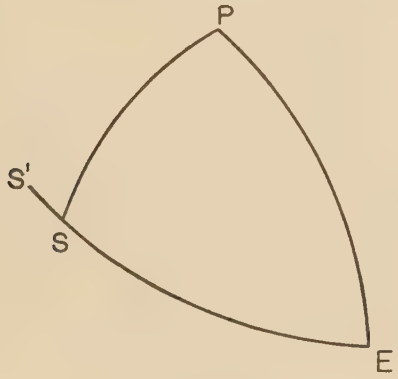


Fig. 1.

one day). The spot groups used were those crossing the central meridian in February, March, April, and August, September, October, of the years 1886–1910. All such spots were included. It was found that the two sets of months gave similar results; they were therefore combined. The ±75° meridians were also grouped together, and the ±65° meridians. Table III. shows the residual angular velocities for different zones of latitude. The unit is degrees per day.

Table III.

Residual Angular Velocities.

Meridian. \ Hel. Lat.	+25	+20	+15	+10	0	−10	−15	−20	−25
Central.	-0.26_{5}	$+0.08_{35}$	$+0.42_{71}$	$+0.22_{84}$	$+0.52_{98}$	$+0.34_{96}$	$+0.36_{99}$	$+0.24_{76}$	-0.05_{29}	$+0.62_{15}$
±65°	-0.97_{7}	-0.89_{53}	-0.26_{88}	-0.13_{111}	$+0.01_{121}$	$+0.05_{136}$	-0.05_{127}	-0.07_{83}	-0.14_{36}	-0.19_{19}
±75	-4.73_{3}	-0.52_{25}	-0.91_{48}	-1.25_{73}	-0.56_{71}	-0.77_{91}	-0.90_{112}	-0.73_{72}	-1.19_{36}	-1.37_{26}

It is evident that there is a marked decrease in the observed angular velocity as the distance from the central meridian increases, but this is better shown by the actual angular velocities obtained by adding 14·18 (the angular velocity in degrees per day of the adopted zero meridian) to the residuals. They are exhibited in the following table, where the weighted means of the residuals for corresponding north and south latitudes have been taken as the residuals for the latitude. The unit is degrees per day. ξ_l denotes the velocity at longitude l.

TABLE IV.

Angular Velocities.

Lat.	ξ_0	$\xi_{\pm 65}$	$\xi_{\pm 75}$
$0°-\pm 10°$	14·61	14·21	13·51
$\pm 10-\pm 15$	14·48	14·10	13·14
$\pm 15-\pm 20$	14·51	14·01	13·37
$\pm 20-\pm 25$	14·20	13·59	13·27
Means	14·45	13·98	13·29

The rate at which the decrease of angular velocity increases as the distance from the central meridian increases should be noticed. For the 10° from $\pm 65°$ to $\pm 75°$ it is about $\frac{3}{2}$ times the decrease from the central meridian to $\pm 65°$. The irregularity of the numbers for ξ_0 and $\xi_{\pm 75}$ in the different latitudes is mainly due to 131 spots having residuals greater than 3°·0. When these were omitted they became more uniform. Table V. shows the values of Table IV. when these spots are not included.

TABLE V.

Angular Velocities, including only those residuals numerically less than 3°·0.

Lat.	ξ_0	$\xi_{\pm 65}$	$\xi_{\pm 75}$
$0°-\pm 10°$	14·57	14·20	14·06
$\pm 10-\pm 15$	14·49	14·04	13·86
$\pm 15-\pm 20$	14·44	14·04	13·80
$\pm 20-\pm 25$	14·20	13·86	14·19

The values of this table have not been included in any subsequent calculations.

Let us consider what effect refraction will have on the angular velocity (fig. 2). If the rays are refracted, a spot will be visible for a short time after it has passed round the actual limb. When it appears to be on the limb at S, its true position will be S′, where the ray to the earth is tangential to the sun's surface. When it reaches A, the actual and apparent spots will be together. Accordingly, the angular velocity of S is always less than that of S′, except at A, where it is the same. Moreover, the difference will

increase at an increasing rate as the limb is approached. The angular velocities of Table IV. fulfil both these requirements, and could therefore be explained by a refraction hypothesis.

Examining the problem of refraction more closely, there appear to be two effects which in practice may be dealt with separately. There is the effect due to bending of the ray, which is analogous to terrestrial refraction, and a second consequence, the magnification of the sun's radius. Although at the centre of the sun the displacement of a spot vanishes, the angular velocity at the point is not the actual angular velocity, for one is referred to the observed sun, and the other to the actual sun. As a

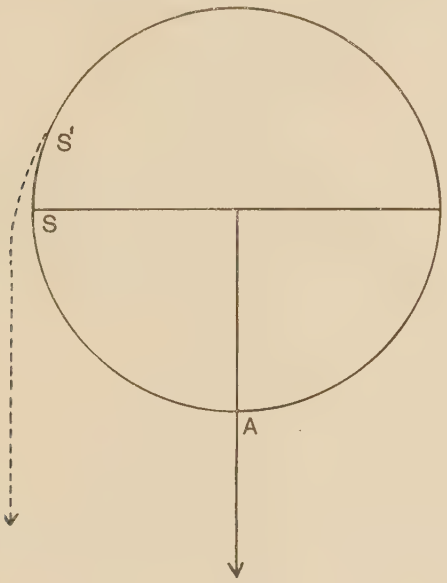


FIG. 2.

rough example, suppose the sun's atmosphere to be homogeneous and its refractive index to be μ. Then it is found that whatever the thickness of the atmosphere, the observed radius is μ times the actual radius. Let r' ξ' be the actual radius and actual angular velocity at the equator, and r the corresponding observed quantities. To an observer situated in the equatorial plane, the observed and actual spots will cross the line of sight with the same velocity when the spot is at the centre of the disc. This gives

$$r\xi = r'\xi'$$

and therefore $\xi' = \mu\xi$ by the above relation between r and r'; that is, the observed angular velocity is $\frac{1}{\mu}$ times the actual angular velocity. This effect, which may be termed the radius effect for

brevity, will preponderate near the centre of the disc, but may be overpowered by the first, or "refraction" effect, as the limb is approached. A formula which enables us to differentiate between the two may be deduced in the following way :—

Let ϕ', l', ${}_{\phi}\xi_{l}'$ be the latitude, longitude, and angular velocity of the actual spot on the actual sun, and let ϕ, l, ${}_{\phi}\xi_{l}$ be the corresponding quantities for the apparent spot on the apparent sun.

Since we neglect the bending of the ray, the velocities of the actual and apparent spots across the line of sight will be the same. This gives

$$ {}_{\phi}\xi_{l}'\, r' \cos\phi' \cos l' = {}_{\phi}\xi_{l}\, r \cos\phi \cos l. $$

And, putting $l = 0$ and consequently $l' = 0$,

$$ {}_{\phi}\xi_{0}'\, r' \cos\phi' = {}_{\phi}\xi_{0}\, r \cos\phi $$

$$ \therefore \quad \frac{{}_{\phi}\xi_{l}}{{}_{\phi}\xi_{0}} = \frac{\cos l'}{\cos l}. $$

Now ${}_{\phi}\xi_{l}$ and ${}_{\phi}\xi_{0}$ are both observed. We may therefore find l', the actual longitude of the spot when its apparent longitude is l.

Moreover, to a sufficient approximation the line of sight is parallel to the central meridian plane, and to the equatorial plane (taking the earth to be in the plane of the equator as an approximation). Therefore

$$ r \cos\phi \sin l = r' \cos\phi' \sin l' $$

$$ r \sin\phi = r' \sin\phi'. $$

Eliminating ϕ', and putting $\sin l / \sin l' = p$, we get

$$ r^2(p^2 \cos^2\phi + \sin^2\phi) = r'^2. $$

If this equation gives the same value of r/r' at a chosen meridian as the value obtained from a meridian close to the central meridian, where the refraction effect is known to be zero, then the refraction effect is negligible at that meridian.

Table VI. exhibits the values of r/r' deduced from the angular velocities of Table IV.

Table VI.

Values of $\frac{\textit{observed radius}}{\textit{actual radius}}$.

ϕ	$l = \pm_{65}$	$l = \pm_{75}$
$0 - \pm 10$	1·0057	1·0055
$\pm 15 - \pm 20$	1·0053	1·0065
$\pm 20 - \pm 25$	1·0067	1·0051
Means	1·0059	1·0057

The means agree fairly closely, but in view of the short distance between the meridians, and the rather irregular appearance of the data, the angular velocities, it would probably be unsafe

to assume from this that the refraction effect is negligible at longitude $\pm 75^\circ$ from the central meridian.

Neglecting the refraction effect, the ratio ξ_l/ξ_0 has been calculated, assuming $r/r' = 1{\cdot}006$. Table VII. shows the ratios for every 10° of longitude from the central meridian. The values apply to the equator, but there is very little change with latitude.

TABLE VII.

Values of ξ_l/ξ_0.

$l=0$	$l=\pm 10$	$l=\pm 20$	$l=\pm 30$	$l=\pm 40$	$l=\pm 50$	$l=\pm 60$	$l=\pm 70$	$l=\pm 80$
1·006	1·006	1·006	1·007	1·008	1·009	1·011	1·014	1·029

It may be asked why it is necessary to assume a refraction effect at all, if the radius hypothesis fits the facts so well. Why should we not assume that the spots are a little distance beneath the photosphere? There are two objections to this simple solution. First, the latitude drift would be unexplained. There would be a small latitude drift in the opposite direction to that found, but to the degree of accuracy employed, it would not enter into the results. Second, from the formula $r \sin l = r' \sin l'$ found above, we find that when $l' = 90^\circ$, $l = 83^\circ\ 54'$. Now, the spot would disappear on this hypothesis when $l' = 90^\circ$. Accordingly, no equatorial spot would ever come within 6° 6′ of the limb, and, as the latitude increased, this premature disappearance would become still more marked. On the other hand, assuming the magnification of the radius to be due to refraction, the latitude drift is explained, and the apparent spot may proceed to the edge of the disc; for in this case it is still visible when the actual spot has passed behind the actual sun, as shown in fig. 2.

The refraction hypothesis also tends to explain the apparent increase of angular velocity in the higher layers of the sun's atmosphere. As we recede from the surface of the sun, both the radius effect and the refraction effect become less marked, consequently the angular velocity approaches its actual value, which we have seen is greater than the velocity at any meridian in the same latitude. Moreover, it might be found to supply a reason for the apparent change of angular velocity with the length of life of a spot, which is one of the conclusions of a paper by E. W. and A. S. D. Maunder on the solar rotation period in *M.N.*, vol. lxv., No. 8. The mean angular velocities taken from this paper are exhibited in Table VIII.

TABLE VIII.

Character of Spot Group.	Mean ξ.
Short spots, observed on six or seven days	14·50
Medium spots, observed on eight, nine, or ten days	14·48
Long spots, observed on eleven, twelve, thirteen, or fourteen days	14·33
Recurring spots	14·29

The "long" spots of this table move almost from limb to limb. The angular velocity deduced from them is therefore the mean of the angular velocities at each meridian. Now, Table VII. shows the rapid decrease in angular velocity near the limb, and in the case of the "long" spots, these diminished velocities will enter once with each spot, and will consequently influence the result considerably. In the case of the "short" and "medium" spots, however, these small angular velocities will only enter with a few spots which happen to be close to the limb. We should consequently expect a greater velocity from these than from the "long" spots, although the actual angular velocity of all is the same. The result for the recurrent spots appears to be opposed to the theory. The angular velocity deduced from the period of a recurrent spot from central meridian to central meridian again is the true angular velocity, which has been found to be the greatest. This is directly opposed to the conclusion of Table VIII. There are, however, two causes which would tend to decrease the theoretical value. First, the initial and final positions of the spot are not necessarily on the central meridian. Consequently, some of the smaller angular velocities will enter into the result. Second, the recurrent spots are found to be situated further from the equator, on the average, than the non-recurrent spots. There are approximately equal numbers of recurrent spots above and below latitude 15°, while the corresponding latitude for recurrent and non-recurrent spots together is $13\frac{1}{2}$°. If from this last we subtract the effect of the recurrent spots mixed with the non-recurrent spots, the difference will be still more marked. Now, the angular velocity decreases with increasing latitude. Consequently, the effect of this will be to decrease the mean angular velocity of the recurrent spots with respect to the non-recurrent spots. It may happen that these two causes together are sufficient to overpower the increase demanded by the theory.

Briefly, the conclusions are as follows:—

(1) An increase of latitude has been found as a spot approaches the central meridian, the increase becoming more marked as the distance of the spot from the centre of the disc increases.

(2) There is found to be an apparent decrease of angular velocity as the longitude distance from the central meridian increases, the rate of decrease increasing more rapidly as the limb is approached.

(3) Both these effects have been explained on the hypothesis of refraction, and refraction has been shown to have a twofold effect, an increase of the sun's apparent radius, and a simple bending of the rays analogous to terrestrial refraction.

(4) Of the two consequences mentioned in (3), it has been shown to be probable that the former preponderates within 75° of the centre of the disc. Assuming this to be the case, the ratio (observed radius)/(actual radius) has been found to be approximately 1·006.

Since the above note was written, my attention has been called to a somewhat similar investigation by Mr. R. C. Carrington,

recorded in *M.N.*, vol. xviii. pp. 169–177. A small refraction effect is found, but the author remarks that the conclusion will probably be considered as entitled to little weight. That the conclusion is indefinite is not surprising, when it is noticed that the data are obtained from the motions of only two spots in longitude and two in latitude. In view of the magnitude of the irregular proper motions, especially those in longitude, it seems clear that to obtain a definite result the mean of a large number of observations must be employed. Moreover, the comparison hypothesis postulates a homogeneous atmosphere. The ground for this is the fairly satisfactory nature of the hypothesis in the case of terrestrial refraction. The problems appear to be very different, however, for there is no "radius effect" when the refraction is caused by the earth's atmosphere, and in the case of solar refraction it has been shown to be probable that this is the effect which preponderates, at least within 75° of the centre of the disc.

Results of the Observations made with the Almucantar of the University of Durham during 1912. By E. H. Hills, F.R.S., and F. C. H. Carpenter.

The following paper contains the results obtained with the Durham Almucantar during a course of observations made during 1912, with a discussion on the errors of the instrument. The description of the instrument and of the methods of observation and reduction were given by Prof. Sampson in two communications (*M.N.*, lx. 572, and lxiii. 338). No further reference will be made to these except only when departures from Prof. Sampson's methods have been made.

The subject matter will be arranged in the following order :—

(1) A short discussion on the probable causes of error in the observations.

(2) A description of the mechanical alterations already made in the almucantar and those proposed for future consideration.

(3) A summary of the observations made during 1912.

(4) General conclusions.

On the Errors of Almucantar Observations.

The errors of the almucantar may conveniently be divided into two portions—those which are common to other astronomical instruments of precision, and those which are peculiar to the almucantar ; we are concerned here solely with the latter.

They are :—

(*a*) Error due to float not taking up exactly the position of equilibrium after disturbance.

(*b*) Change of apparent zenith owing to temperature changes in mercury and float.

(*c*) Errors of observation owing to unsteadiness of the telescope; this being freely floating at the moment of observation, cannot be fixed or clamped.

All the above are subject to multiplication by a certain factor due to the obliquity of the transits over the wires and the variation of the apparent angular velocities of the stars.

This last consideration must be continuously borne in mind, so it will be convenient to set it out at once in tabular form. We have:—

$$\frac{dt}{dz} = \frac{1}{15\cos\delta\sin t}; \quad \frac{dt}{d\delta} = \frac{1}{15\cos\delta\tan t}; \quad \frac{dt}{d\phi} = \frac{2}{15\sin 2\phi\tan t}.$$

Therefore, if we assume firstly time of observation and star places correct and that we are computing latitude; secondly, assuming latitude correct, that we are computing star-places, then:—

δ of Observed Star.	Effect of 0·1 sec. error in Transit.	Effect of 1″·0 error in Z.D.
	$d\phi =$	
+20°	0″·09	0″·51
40°	0·86	0·93
60°	1·72	2·48
	$d\alpha =$	
20°		0″·54
40°	dt	0·12
60°		0·14
	$d\delta =$	
20°	0″·19	1″·01
40°	1·31	1·52
60°	1·83	2·64

(*a*) and (*b*) These can be conveniently dealt with together.

All the accumulated evidence goes to show that the float regains its equilibrium position after disturbance, provided it is subject to no sort of constraint, with a very high degree of precision, but that there is often a continuous change in the equilibrium position presumably due to temperature change in the mercury.

A large number of level observations have been made using the transit level of the Observatory placed on the telescope in varying azimuths and then taking a set of readings with the float disturbed between each. Four examples of the sort of results obtained may be given as specimens, the first two being selected from among those taken when temperature was apparently stable, the second two when it was changing.

Level Observations.
1913, Jan. 24.

Object End.	Eye End.
24·5	16·0
24·5	16·0
24·5	16·0
24·5	16·0
24·5	16·0
24·4	16·0
24·4	15·9
24·4	16·0
24·4	16·0

1 Division = 1″·5.
1913, Jan. 25.

Object End.	Eye End.
27·2	16·2
27·2	16·2
27·2	16·2
27·2	16·2
27·2	16·2
27·1	16·2
27·2	16·2
27·2	16·2
27·2	16·2
27·2	16·2

1913, Feb. 11.

Object End.	Eye End.	
22·3	17·4	
22·7	17·1	
22·9	16·8	
23·0	16·6	Change
23·1	16·5	1″·65.
23·1	16·3	
23·2	16·0	
23·1	16·0	

1913, Feb. 11.

Object End.	Eye End.	
21·0	16·2	
21·0	16·2	
21·0	16·2	
20·9	16·2	
20·9	16·2	
20·8	16·3	Change
20·8	16·3	0″·75.
20·7	16·3	
20·6	16·4	
20·6	16·4	
20·4	16·6	
20·4	16·6	

Results of star observations seem to show that this temperature change is by far the most serious source of error that we have to contend with. The apparent zenith often alters by as much as 3″ or 4″ in the course of a night's observations; on one night (Oct. 12) a change of 1″·57 per hour was found. In so far as this change is a linear function of the time, it can be and has been eliminated in the reductions; cases where this has been done will be seen in the summary of the 1912 observations below. Even then there remain, however, outstanding irregularities of considerable magnitude. It seems a fair conclusion that the "unexplained residual" found by Prof. Sampson, and set out below for the 1912 observations, is partly a temperature error.

Apart from this continuous change in zenith there is sometimes found a sudden shift, apparently due to some friction at the stops which prevent lateral movement of the float. Much consideration has been given to the question as to whether a mechanical improvement could not be made here and some form of lateral constraint applied which would be frictionless. It is

not difficult to devise a frictionless constraint, but it would be at the expense of introducing a disturbing torque about a horizontal axis due to wind pressure on the telescope, obviously most undesirable.

(*c*) The unsteadiness of the telescope at the moment of observation, owing to the fact that it cannot be clamped, may undoubtedly cause serious errors, but it is difficult to arrive at any numerical estimate of their amount. This quantity could be ascertained by a comparison of the probable error of a wire interval derived from observations made with the instrument floating with that derived from a set of observations made by the same observer with the float clamped. This would be laborious and has not been done.

While the almucantar has thus certain serious disadvantages as compared with other instruments fulfilling the same function, *e.g.* the transit circle, it has the apparent advantages that the circle of reference is a fixed one and that no reading of graduated circles is required.

The first-named advantage is, as has been seen, very illusory, since as a matter of fact the almucantar circle is by no means fixed but varies considerably from hour to hour. Even, however, assuming that it were fixed, that is to say that there were no change due to any change in state of the mercury or float, the almucantar would have no advantage in accuracy unless we could assume that a mass of iron floating in mercury will take up its position of equilibrium with a greater degree of precision than it is possible to read a spirit-level. We are not entitled to make this assumption. As to the question of divided circles, while the fact that they are not required unquestionably saves time and trouble in observing, it cannot be contended, seeing the perfection of circle graduation that has been attained and the completely efficient methods that have been evolved for determining their errors, that this imparts any increased accuracy to the results.

It will thus be seen that almucantar observations are liable to sources of error which are not common to other instruments, and that such advantages as they have lie hardly, if at all, in the direction of increased precision. We shall be justified in concluding that it is highly improbable that the almucantar can ever surpass or even equal the transit circle as an instrument of precision.

This conclusion is borne out by the results of the observations at Durham; but it is possible that the part of the errors due to temperature changes might be largely eliminated by a mechanical alteration in the instrument. This will be dealt with in the next section.

Mechanical Improvements in the Instrument.

Certain minor improvements were effected in the Durham instrument before observations with it were resumed in 1912. The

most important of these was that the iron float, which had been left in a rough state direct from the casting, was finely polished on the bottom and sides to reduce skin friction between it and the mercury. This appears to have been distinctly beneficial. Much consideration was given to the question as to whether it was not advisable to adopt some more rigorous precautions to exclude the possibility of the occurrence of air-bubbles under the float, the first idea being to have a new float made with a sloping bottom, so that by tapping it any bubbles would rise to the surface. Experiments made with glass plates showed, however, that even when tilted to an angle of 30° or more the plate would stand a considerable amount of shaking and knocking before bubbles were dislodged, so that this idea had to be given up. These experiments at the same time showed that even if there were any bubbles it was almost impossible that any movement of the float made in the process of observing could shift their position, so that their presence was of altogether minor importance. It was also pointed out by Messrs Cooke that if before the float was placed on the mercury, a sheet of paper, or, better, muslin, was laid on the surface and then, after the float was submerged, was steadily drawn away, it would carry with it any bubbles there might be. Experiments made as before with a glass plate showed the complete effectiveness of this device. The lighting apparatus was also altered. Formerly this had been done by little pea lamps at the eye end, necessitating wires attached to the telescope. This it is desirable to avoid, so the eye-end lighting was removed and an illuminated opal plate at the object-glass end substituted. The telescope is now, except for the contact at the six stops, in free flotation at the time of observation.

The polishing of the float introduced one disadvantage in that the damp air of Durham rusted the exposed surface. This was remedied by keeping a layer of oil on the top of the mercury ; at first paraffin was used, but as this was found to creep over the top of the float and over the telescope, a mixture of sperm oil and paraffin was tried which has proved quite satisfactory.

The principal improvements that now remain to be effected are :—

Firstly, a more efficient wind screen. The moving part is now surrounded by a cage of perforated zinc shielding it from the worst of the wind pressure, but enough remains to interfere seriously with observation. As in any case the object end of the telescope must remain exposed, and as any wind pressure on this exerts considerable leverage on the float, it does not seem possible to eliminate the wind disturbance altogether. A solid screen might, however, be advantageous.

Secondly, an improvement in the stops preventing lateral movement. This has been already referred to. Some alteration is most desirable, but no practicable suggestion has yet offered itself.

Thirdly, some arrangement for disturbing the mercury in the trough so as to ensure equable temperature distribution. There seems little doubt that in this respect the design of the Durham

instrument is faulty. To move the telescope in azimuth the whole trough is rotated, whereas if the trough were circular instead of rectangular and the floating part only turned, the mercury would be thoroughly mixed up at each new star. It is not possible to alter this short of an entire reconstruction of the instrument.

A circular trough would present the further advantages that the perimeter at the line of flotation, and consequently the skin friction, would be a minimum, and also that the whole instrument would be more nearly symmetrical.

Observations made during 1912.

Altogether observations have been made on thirty-five nights, of which four have been rejected.

The reductions have followed the methods devised by Prof. Sampson, except that a "second solution" has been applied to eliminate systematic errors such as error in stars' tabular place, or want of horizontality in individual wires, which should be more or less constant for the same star night after night. Whenever, therefore, three or more observations could be got of the same star, the mean of the residuals was taken; this was applied and the equations solved again, with the result that some of the apparent error is generally eliminated.

Any continuous change in z during the night is allowed for.

In cases where a sudden change of z has occurred the stars have been solved in two batches.

Stars have been rejected which were certainly in error, owing presumably either to sticking of the telescope or to excessive vibration, but a consecutive set of stars has invariably been retained for the solution.

Date. 1912.	No. of Stars Observed.	Mean v. 1st Sol.	Mean v. 2nd Sol.	10z 1st Sol.	10z 2nd Sol.	$\Delta\phi''$ 1st Sol.	$\Delta\phi''$ 2nd Sol.	Remarks.
Mar. 5	24	±·030	±·022	+ ″·317	+ ″·306	− ″·295	− ″·260	
6	25	±·030	±·022	− ·965	− ·926	− ·240	− ·298	2 stars rejected from solu tion and mean v.
11	26	±·031	±·014	− 1·089	− 1·133	+ ·129	+ ·171	26 stars observed. The fir 19 only used in solutio The last 7 gave a mean 1 equal to 0″·38.
13	24	±·032	±·022	+ 2·191	+ 1·262	− ·176	+ ·005	Change in 10z of +″·56 p hour adopted.
Apr. 11	28	±·029	...	+ 1·834	...	+ ·181	...	Only 17 stars used in solutio Telescope very unstead Mean 10z for stars 1–4 = +3″·28 5–15 = +1·81 16–20 = +0·83 21–28 = +1·94
16	32	±·041	±·024	+ 2·585	+ 2·602	− ·213	− ·167	Only 27 stars used in solutio Mean 10z for stars 6–10 = +1″·01.
20	27	±·047	±·043	+ 5·797	+ 5·819	− ·252	− ·320	Mean 10z for stars 2–6 +6·88. These have be included in mean v.

Date. 1912.	No. of Stars Observed.	Mean v. 1st Sol.	Mean v. 2nd Sol.	$10z$ 1st Sol.	$10z$ 2nd Sol.	$\Delta\phi''$ 1st Sol.	$\Delta\phi''$ 2nd Sol.	Remarks.
pr. 22	32	±·049	±·025	+ 6″·083	+ 5″·009	−″·463	−″·398	Change of +″·41 per hour adopted to $10z$.
23	20	±·042	±·020	+ 2·272	+ 2·352	−·235	−·152	
24	30	±·032	±·014	+ 1·314	+ 1·412	−·040	−·099	19 stars used in solution. Mean $10z$ for stars 1–6 = +″·56; 7–25 = +1·41; 26–30 = +2·52
ay 13 [	26	...	...	− 4·798	...	+·183	...]	Change in z between stars 8 and 9 and again between stars 23 and 24.
	1–8	...	±·023	...	+ 6·071	...	−·148	
	9–23	±·034	...	+ 4·382	...	−·139	...	
pt. 12	18	±·026	±·019	+15·078	+15·010	−·513	−·466	
14	11	±·028	...	+16·183	...	−·491	...	
19	16	±·064	±·049	+13·442	+13·386	−·789	−·848	Change of z between stars 14 and 15.
21	35	±·042	±·036	+14·404	+13·676	−·728	−·660	Hourly change of $10z$ = +″·32 adopted.
22	28	±·035	±·030	+14·192	+14·218	−·667	−·435	One star omitted from solution and mean v, z altered.
24	24	±·039	±·027	+ 1·691	+ 1·659	−·589	−·605	
t. 3	27	±	...	...	...	...	...	
	1–17	±·051	±0·32	+ 2·045	...	−·801	−·337	
	18–27	±·025	±·022	− ·800	...	−·702	...	
6	15	Rejected.						Telescope probably sticking.
7	26	±·049	±·033	+ 2·194	+ 2·125	−·441	−·388	
9	13	Rejected.						
10	29	±·050	±·041	+ 2·089	+ 2·119	−·433	−·429	
12	16	±·025	±·016	+ 2·991	+ 1·412	−·417	−·538	Hourly change of +1″·57 in $10z$ adopted.
20	19	±·039	±·030	− 1·599	− 1·601	−·368	−·390	1st star rejected from solution and mean v.
25	35	±·037	±·030	− 0·236	− 0·616	−·590	−·562	Hourly change of +″·30 in $10z$ adopted.
30	13	±·045	±·028	+ 0·231	+ 0·164	−·433	−·425	
7. 3	9	Rejected.						
17	16	...	...	...	...	...	...	
	1–8	±·013	±·013	− 1·322	− 1·333	−·277	−·203	
	9–16			− ·027	− ·173	−·385	−·308	
22	20	±·032	±·025	+ 4·573	+ 4·457	−·017	+·146	Last 12 stars used in solution. Interval of 1 h. 40 m. between stars 8 and 9 owing to clouds. Bad night. Very windy.
25	9	Rejected.						
30	25	...	...	...	...	...	...	The first 7 stars were omitted from solution as there was a change in z between stars 7 and 8.
	8–16	±·022	...	+ 2·052	...	−·221	...	In the 1st solution the remaining stars were solved up in 2 lots of 9 each.
	17–25			+ 2·726	...	−·663	...	

Date. 1912.	No. of Stars Observed.	Mean v. 1st Sol.	Mean v. 2nd Sol.	$10z$ 1st Sol.	$10z$ 2nd Sol.	$\Delta\phi''$ 1st Sol.	$\Delta\phi''$ 2nd Sol.	Remarks.
Nov. 30	8–25 less 17 18–19	...	±·022	...	+ 2·169	...	−·238	In the 2nd solution 15 sta were used.
Dec. 2	21	±·039	...	+ 0·365	...	−·482	...	
12	20	±·029	±·016	+ 6·348	+ 6·261	−·741	−·574	Change in z between star and 7. Last 14 stars used for so tion.
16	18	±·031	±·026	+ 5·279	+ 5·310	−·343	−·447	Last 2 stars omitted fr solution and mean $10z = +7''.3$ for these t stars.
18	27	±·039	±·024	+ 4·405	+ 3·379	−·362	−·201	Hourly change in z. +" adopted.

The "mean v" column is the mean of the residuals, irrespective of sign.

The "$10z$" column is collimation error from the adopted colatitude circle, positive when telescope is pointing below it and negative when telescope is pointing above it.

The "$\Delta\phi$" column is the correction to latitude as found from the night's observations.

We can calculate the v or "unexplained residual" in the same way as before. It would be redundant to set this out in full for every night's observations, but as a specimen of the sort of results obtained it is given below for every fifth night:—

Unexplained residuals "v" in individual observations.

1912, March 5.			1912, April 16.			1912, May 13.		
α Lyncis	E	+01	8 Can. Ven.	E	−02	10 Leo. Min.	W	+02
43 Cass.	W	−01	α Drac.	E	−03	τ Herc.	E	+02
4 H. Drac.	E	+06	10 Camel.	W	+02	ϵ_2 Boötis	E	+04
τ Persei	W	+04	o Aurigæ	W	+01	ι Urs. Maj.	W	00
γ Urs. Maj.	E	+02	α Can. Ven.	E	[−09]	o Urs. Maj.	W	−04
c Persei	W	+01	η Urs. Maj.	E	[−·10]	Groom. 1374	W	+02
χ Urs. Maj.	E	+01	Groom. 848	W	[+08]	β Coronæ	E	−04
ϵ Leonis	E	−05	α_2 Gem.	W	[+05]	χ Drac.	E	−01
μ Aurigæ	W	−01	γ Urs. Min.	E	[−09]	β Drac.	E	+03
η Aurigæ	W	−02	ι Drac.	E	−02	θ Urs. Maj.	E	+01
ϵ Aurigæ	W	00	8 Lyncis	W	+03	μ Urs. Maj.	W	−01
ϵ Urs. Maj.	E	+01	22 Camel.	W	00	γ Drac.	E	+03
4 Urs. Min.	E	−03	ϵ Leonis	W	+01	σ_2 Urs. Maj.	W	+07
α Aurigæ	W	−01	A Drac.	E	+04	τ Drac.	E	+03
θ Aurigæ	W	−02	θ Drac.	E	+05	ν Urs. Maj.	W	−04
8 Can. Ven.	E	+06	β Boötis	E	+01	δ Drac.	E	00
α Drac.	E	−02	31 Lyncis	W	−05	ϵ Herc.	E	+04
β Aurigæ	W	−02	11 Boötis	E	+04	ϵ Drac.	E	−10
o Aurigæ	W	+03	α Lyncis	W	00	31 Coronæ	W	+06
α Can. Ven.	E	00	ν_1 Boötis	E	+05	Groom. 1586	W	+02

Unexplained residuals "v" in individual observations—continued.

1912, March 5.

η Urs. Maj.	E	+04
ρ Gem.	W	00
α_2 Gem.	W	−06
θ Boötis	E	00

p.e. = ± ·020

1912, April 16.

μ Boötis	E	+04
10 Leo. Min.	W	+02
τ Herc.	E	−04
ϵ Boötis	E	00
ι Urs. Min.	W	+02
o Urs. Maj.	W	00
Groom. 1374	W	−02
β Coronæ	E	−04
χ Drac.	E	−01
β Drac.	E	00
θ Urs. Maj.	W	−01
μ Urs. Maj.	W	+08

p.e. = ± ·021

1912, May 13.

κ Cygni	E	−03
76 Drac.	E	−03
β Urs. Maj.	W	+01

p.e. = ± ·026

1912, September 22.

43 Cass.	E	00
50 Cass.	E	+02
o Cass.	E	+01
β Pegasi	E	+02
Groom. 2377	W	−13
ϵ Cass.	E	+04
ζ Urs. Min.	W	00
55 Cass.	E	−02
β_1 Lyræ	W	+01
γ Lyræ	W	+05
α Lyræ	W	+05
μ And.	E	−08
α And.	E	−10
ζ Drac.	W	+03
δ And.	E	+01
β And.	E	00
θ Lyræ	W	+01
ϵ And.	E	−02
ϵ Urs. Min.	W	00
τ Persei	E	00
θ Persei	E	−02
32 Vulpec.	W	00
Groom. 848	E	−04
κ Cygni	W	+01
θ Cygni	W	−05
χ Drac.	W	−06
β Persei	E	[+14]

p.e. = ± ·030

1912, October 12.

δ And.	E	−02
β And.	E	−01
θ Lyræ	W	+01
ϵ And.	E	00
ϵ Urs. Min.	W	+01
τ Persei	E	−03
θ Persei	E	−02
32 Vulpec.	W	−02
β Trian.	E	00
κ Cygni	W	+02
θ Cygni	W	−01
χ Drac.	W	−02
16 Peg.	W	+04
β Persei	E	+01
ρ Persei	E	+02
9 Camel.	E	+02

p.e. = ± 013

1912, November 22.

δ Drac.	W	+02
61 Cygni	W	+02
ν Cygni	W	+04
α Cygni	W	+01
4 Camel.	E	−05
10 Camel.	E	+01
τ Drac.	W	−03
α Arietis	E	00
41 Arietis	E	+04
ϵ Drac.	W	−04
η Peg.	W	−02
ξ Persei	E	−02

p.e. = ± 020

These results may be compared with Prof. Sampson's in 1901–2 by taking the arithmetic mean of the probable errors thus:—

1901–2	1912
45 nights observations	31 nights observations
mean p.e. = 0″·36	mean p.e. = 0″·23

In a similar way we can tabulate the residuals of individual stars. Here again it is redundant to put them all down, so we take every tenth star on the list to give a fair sample of the results:—

Residuals of Individual Stars.

α And.		ζ Persei.		α Drac.		γ Drac.	
E		E		E		E	
Sept. 12	+06	Oct. 20	−01	Mar. 5	−04	April 22	−04
,, 14	−02	Dec. 16	+06	,, 6	+03	May 13	+03
,, 19	−01	,, 18	+07	,, 11	−03		
,, 22	−07			April 16	−04		
,, 24	−03					κ Cygni.	
Oct. 3	00	o Aurigæ.				W	
,, 7	+04	E				Oct. 20	+05
,, 10	+03	Dec. 16	+03	ζ Urs. Min.		Dec. 16	+02
,, 25	−01	,, 18	−08	W		,, 18	−01
Nov. 17	00			Sept. 12	−02		
50 Cass.		ε Leonis.		,, 19	−04	η Peg.	
E		E		,, 21	−09	W	
Sept. 14	−01	Mar. 5	−08	,, 22	−03	Oct. 20	+05
,, 22	−05	,, 6	−02	Oct. 3	−06	Nov. 22	−01
,, 24	+01	,, 11	−03	,, 7	−05	,, 30	−01
Oct. 3	−13	,, 13	00	,, 10	00	Dec. 16	+06
,, 7	−07			,, 25	−03	,, 18	−01
,, 10	−09			Nov. 17	−01		
,, 25	−09						

By comparing these residuals with the tables given at the beginning of this paper we can see the sort of effect they would have upon a determination of star-places or of latitude.

To illustrate this more clearly we may take four stars and calculate the residuals on the assumption that the time of observation and the latitude are correct, and that the residuals are in the form of correction to the tabular R.A.; and four other stars, and calculate the residuals in the form of correction to assumed latitude, on the assumption that time and star-places are correct. The stars are selected quite at haphazard, except that for

latitude determination stars of low declination must have the preference.

Residuals in the Form of Correction to Tabular R.A.

55 Cass. E		θ Persei. E		γ Lyræ. W		β_1 Lyræ. W	
	s		s		s		s
Sept. 12	+0·02	Sept. 14	−0·04	Sept. 12	−0·01	Sept. 12	−0·04
14	− 03	21	− 01	14	+ 06	14	− 02
19	+ 07	22	± 00	19	− 10	19	− 10
21	+ 02	24	+ 05	21	− 01	21	− 06
22	− 05	Oct. 3	+ 07	22	+ 09	22	± 00
Oct. 3	+ 02	10	+ 08	24	± 00	24	± 00
7	− 05	12	− 04	Oct. 3	− 05	Oct. 3	± 00
10	− 14	25	− 07	7	+ 08	7	+ 08
25	− 02	Nov. 17	+ 02	10	+ 11	10	+ 06
Nov. 17	± 00			25	+ 14	25	+ 06
				Nov. 17	+ 01	Nov. 17	+ 02

Residuals in the Form of Correction to Assumed Latitude.

ε Androm. E		α Arietis. E		β_1 Lyræ. W		32 Vulpec. W	
	″		″		″		″
Sept. 12	−0·1	Oct. 20	−0·4	Sept. 12	+0·2	Sept. 12	+0·1
21	−0·1	30	−0·2	14	−0·2	21	+0·1
22	+0·1	Nov. 22	−0·2	19	−0·6	22	−0·2
24	−0·1	30	0·0	21	−0·4	24	−0·1
Oct. 3	−0·1	Dec. 12	−0·1	22	0·0	Oct. 3	−0·3
7	+0·5	16	−0·1	24	0·0	10	−0·1
10	+0·1	18	−0·2	Oct. 3	0·0	12	−0·2
12	+0·3	W		7	+0·5	25	+0·3
25	+0·3	Dec. 2	−0·2	10	+0·5	Nov. 17	+0·1
Nov. 17	−0·1			25	+0·4		
				Nov. 17	+0·2		

General Conclusions.

On the whole the results are not very encouraging. The two sources of error peculiar to any floating instrument—the temperature gradient in the liquid and the unsteadiness of the telescope—are doubtless capable of reduction, but their elimination seems beyond the region of possibility.

It is, moreover, difficult to see exactly what function the almucantar can fulfil, or what are the observations for which it is specially adapted, apart possibly from observations in the zenith. It certainly cannot determine relative right ascensions with the

precision of the transit circle, or latitudes with the precision of an altazimuth of quite moderate size. It is unsuited for fundamental star-places, *i.e.* for observations of the same star east and west of the meridian, because the stability of the instrument cannot be trusted for any length of time, and for those stars of which the two transits are at short intervals the obliquity of the path across the wires becomes excessive.

It does not appear that there is any systematic error to which transit circle observations are liable from which almucantar observations are free,* so that there is no justification on this ground for the use of an instrument of decidedly inferior accuracy.

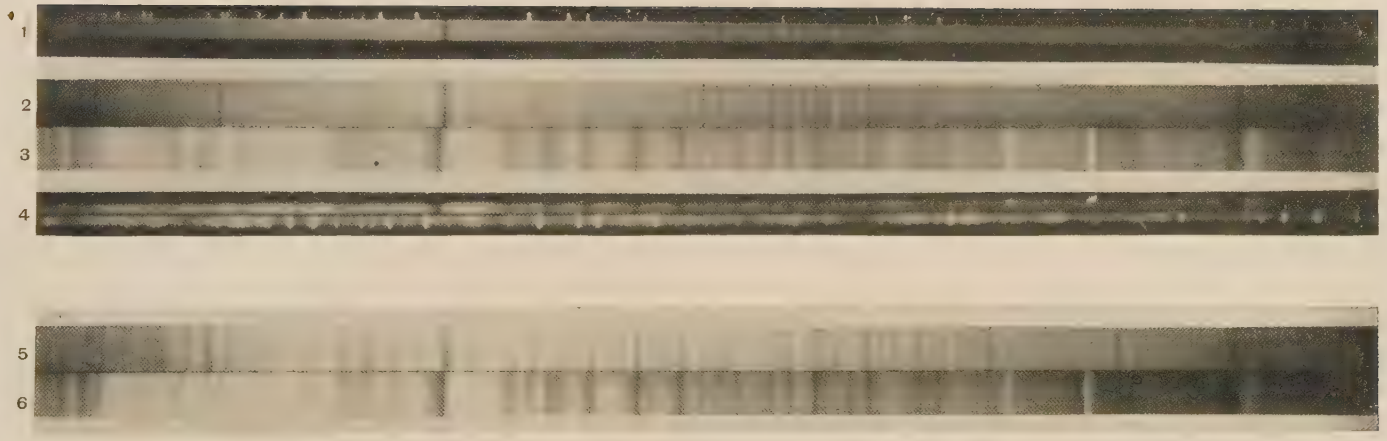
Observations with the Durham instrument will be continued for the present, and if any marked improvement is arrived at a further report will be made to the Society.

On Enhanced (Spark) Lines in the Early Spectrum of Nova Geminorum No. 2. By H. F. Newall, M.A., and F. J. M. Stratton, M.A. (Plate 11.)

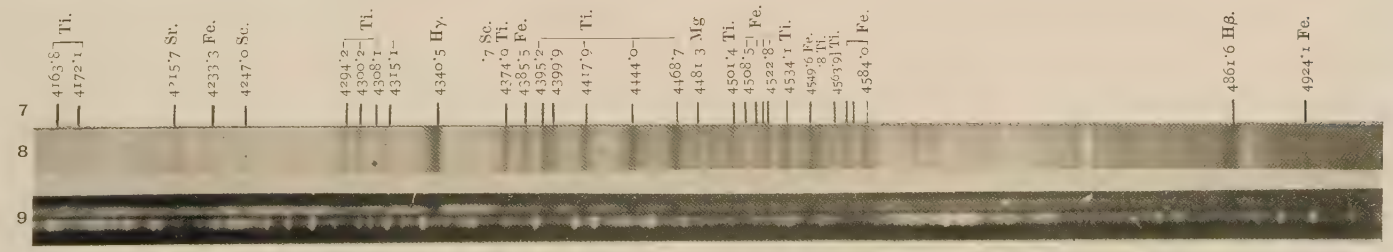
A detailed study is now being made of the early spectra of Nova Geminorum No. 2 obtained at Cambridge in March and April 1912. Inasmuch as varying views have been put forward by other observers on one or two questions of interest, a preliminary note on some salient points in our work seems advisable.

Professor E. C. Pickering (*H.C.O. Circ.*, No. 176) announced that the earliest spectrum (March 13) was of type F 5. Mr. R. H. Curtiss of the Michigan Observatory (*Ap. J.*, xxxv. 233) failed to find in his spectrum of the same date the typical lines of the F 5 spectrum. The explanation lies probably in the fact that the objective-prism photograph taken at Harvard did not reveal the marked displacement of all these lines towards the violet. A spectrum containing so many lines as appear in Procyon does not lend itself readily to the identification of lines with large velocity shifts. In our earliest study of the spectrum, we came to the conclusion (*Observatory*, 1912, p. 174) that the spectrum of the Nova can only be regarded as similar to the F 5 type, if the absorption lines in the Nova are displaced to the violet by about 10 tenth-metres. A comparison with the spectrum of α Cygni has been more definitely fruitful of results. The relative paucity of the lines and their marked grouping have made the resemblance of the two spectra in many details more striking and convincing. We have been able, from measurements of our slit spectrograms with com-

* [Professor Sampson points out that this is not strictly correct. The fact that stars widely apart in R.A. are observed within a short period of time may help to eliminate errors to which observations of consecutive transits over the meridian are liable.—E. H. H. and F. C. H. C.]



1. α Cygni with Fe Comparison. 2. α Cygni widened. 3. Nova widened. 4. Nova with Fe Comparison.
5. γ Cygni widened. 6. Nova widened.



7. Computed positions of displaced enhanced lines. 8. Nova widened. 9. Nova with Fe Comparison.

NOVA GEMINORUM No. 2.

parison spectra, to confirm for the absorption spectrum of the present Nova the close agreement with the spectrum of α Cygni which Sir Norman Lockyer found at South Kensington for the bright-line spectrum of Nova Persei No. 2 (*M.N. R.A.S.*, vol. lxi., Appendix [17]). The identification has been made on the assumption that the strong absorption lines in the Nova are all displaced towards the violet with the dark hydrogen line Hγ, the displacement varying directly as the wave-length.

In the accompanying plate the spectrum of α Cygni is shown side by side with the Nova spectrum, the two spectra being so arranged as to bring a group of strong absorption lines in the two spectra into coincidence. Owing to the regular increase in linear dispersion in prismatic spectra, the velocity displacements are greater in the violet than in the blue. Accordingly, if the spectra of two stars photographed with a given spectrograph are juxtaposed, so as to give coincidence at one line, coincidence at all other lines is only possible when the velocity displacements are equal. The displacements of the hydrogen absorption lines in the Nova on 1912 March 15 are large; they vary with λ, and correspond to a velocity of approach of about 550 km./sec. Accordingly when the spectrum of α Cygni is put side by side with that of the Nova, the criterion for similarity of origin of various lines in the two spectra is in truth a gradually increasing discordance of comparable lines, as we pass in either direction from the line arbitrarily chosen for exact coincidence in juxtaposition. Such a discordance may be easily seen in the accompanying plate. Measures actually made of the absorption lines in the Nova spectrum show that their wave-lengths agree very closely with the calculated values of the star lines displaced proportionately with the centre of the Hγ absorption. In the plate is given a photograph of the Nova spectrum against which are plotted the calculated displaced positions of many enhanced metallic lines found in α Cygni (or in γ Cygni).

Other writers (Schwarzschild, Furuhjelm, W. E. Curtis), have already announced the identifications with enhanced metallic lines; the purpose of this note is primarily to emphasise the close resemblance of the spectrum of the Nova on 1912 March 15 to that of α Cygni in the region photographed at Cambridge. Of the 65 lines of intensity 2 or more given by Miss Maury (*H.C.O. Ann.*, vol. xxviii., pt. i. table vii.) for α Cygni between Hβ and λ 4063, no less than 50 are well identified in the displaced absorption spectrum of the Nova. The differences from α Cygni revealed by the comparison of intensities (see the accompanying plate), and also by a study of the lines missing in either spectrum, show that the Nova is intermediate in type between α Cygni and γ Cygni.

Of the 15 α Cygni lines not found in the Nova, 7 are weak or absent in γ Cygni. The 5 strong lines in the Nova weak or absent in α Cygni are present in γ Cygni, and are strong in 4 out of the 5 cases. The spectrum of γ Cygni, which resembles the chromosphere reversed, has of course many more strong lines than have

been found in the Nova; thus there are in our region 60 lines strong in γ Cygni and absent or weak in α Cygni, and only 4 of these are found in the Nova. Adopting the step system of the Harvard notation, the spectrum of the Nova on 1912 March 15 between Hβ and Hδ might be described as $\frac{2}{10}$ of the way from α Cygni to γ Cygni, or briefly as $\alpha 2\gamma$. (The spectra of α Cygni and γ Cygni are given in the *R.H.P.* as A2 and F 8*p* respectively.)

The elements represented by their enhanced spark lines are titanium (very strongly) and iron certainly; vanadium, scandium, and strontium most probably; and with much less certainty chromium and zirconium.

Radium, nitrogen, oxygen, silicon, and helium are not found in the spectrogram of March 15, either undisplaced or displaced along with the hydrogen, titanium, and iron.

With regard to the two bands so frequently identified with the helium lines at λ 4922 and λ 5016, the evidence points strongly to their being due to the enhanced iron lines at λ 4924, λ 5018. They lie rather near the limit of good focus of the Cambridge plate, and the faint intensity makes measures on the edges of the absorption bands difficult; but the evidence from these measures and from the presence and absence of other lines in the Nova is in favour of the enhanced iron identification. This is supported by the comparison with α Cygni, and by the persistence in later plates of the Nova of bright bands here and in the positions of other enhanced iron lines which are strong in α Cygni. In this we agree with the views of Sir Norman Lockyer as to the sources of the corresponding lines in Nova Persei.

1913 *March* 10.

A Possible Extension of the Spectrum of Hydrogen. By J. W. Nicholson, M.A., D.Sc., Professor of Mathematics in the University of London.

The recent experiments of Professor Fowler have demonstrated that, in important respects, hydrogen conforms in spectral behaviour with other elements. In particular, Rydberg's theoretical deduction of its Principal series has been confirmed in the laboratory. But that there is something peculiar in the hydrogen spectrum which distinguishes it from the known spectra of other substances, is at the same time shown by the discovery of a new ultra-violet series, tending to the same limit as the Principal series. The variable part of this new series is practically identical with that of the Sharp series found by Pickering.

No other element is at present known to show a second Principal series of this type, nor, in fact, of any type which tends to the same limit as the ordinary Principal series. That such series do not exist in other elements is not of course proved by their absence in terrestrial spectra—especially in view of the special conditions

under which the ordinary Principal series of hydrogen has been obtained after a long search in the laboratory.

But, on the other hand, these conditions, in Professor Fowler's experiments, have produced both the Principal series strongly at the same time, and the argument is suggested that other elements would, if they possessed two Principal series, show them simultaneously. If helium and parhelium possessed a second series, some of the lines would necessarily be in the visible spectrum, for their Principal series tend to the limits $\lambda = 3121{\cdot}8$, $2600{\cdot}6$— and, except for two lines in the infra-red, their spectra have been completely fitted into the more usual series.

Thus there is a strong possibility that the spectrum of hydrogen is exceptional, and it is instructive to examine the possibility of other series which it may show—leaving out of account the vacuum tube or secondary spectrum whose origin is still a matter of dispute. The accuracy of the Balmer formula

$$\lambda = \lambda_0 \frac{m^2}{m^2 - 4}$$

and of the formula for the Pickering lines

$$\lambda = \lambda_0 \frac{(m + \tfrac{1}{2})^2}{(m + \tfrac{1}{2})^2 - 4},$$

suggests the possibility of other lines given by

$$\lambda = \lambda_0 \frac{(m \pm \tfrac{1}{3})^2}{(m \pm \tfrac{1}{3})^2 - 4}$$

where $\lambda_0 = 3647$. If these lines have an existence, it is natural to expect them under conditions which produce the Pickering lines strongly. Of the various stars which show the Pickering lines, the class of Wolf-Rayet stars may be selected as the most favourable for this examination. Campbell has given a table of the centres of the Wolf-Rayet bands, from which the following is an extract, containing all those marked as strong.* The intensities are in two divisions, denoted by (1) and (2), and the Pickering lines are very conspicuous:—

λ.	Identification.	λ.	Identification.
5877 (1)	D_3, helium	4517 (1)	...
5813 (2)	...	4509 (2)	...
5693 (2)	...	4504 (1)	...
5593 (1)	...	4480 (1)	[4481] Nova Aurigæ
5472 (1)	...	4473 (1)	Helium
5412 (2)	H (Pickering)	4466 (1)	...
4862 (2)	Hβ	4442 (2)	[4445] Nova Aurigæ
4688 (2)	H (Rydberg)	4389 (1)	Helium
4652 (2)	Nova Persei	4341 (2)	Hγ
4636 (1)	...	4102 (2)	Hδ
4626 (1)	[4629] Nova Aurigæ	4063 (1)	[4067] Nova Aurigæ
4541 (2)	H (Pickering)	4023 (1)	Helium [4026]?

* "The Wolf-Rayet Stars," *Astronomy and Astrophysics*, vol. xiii. p. 448.

The correspondences detected by Campbell with centres of strong Nova bands are also indicated in the table, and also a strong correspondence with a line in the later stages of Nova Persei.

Eight important lines remain which exhibit no certain relation to any other type of spectrum, and which seem to be peculiar to these stars.

Consider now the simple formula

$$\lambda = 3646 \frac{m^2}{m^2 - 9}$$

where $m = 5, 7$. This gives the lines $\lambda = 5696{\cdot}5, 4466{\cdot}3$, which are coincident, within the experimental degree of error in the two regions of the spectrum, with two of these eight lines. The degree of possible error is found by an inspection of the measurements of the hydrogen lines.

Accurate measurements of the Balmer series indicate that the true hydrogen limit is $\lambda_0 = 3647{\cdot}14$, and the limit of the Pickering series, derived from Professor Fowler's measurements, is $\lambda_0 = 3645{\cdot}6$. The values of Rydberg's constant μ in the Sharp series is not exactly $\frac{1}{2}$, but $0{\cdot}4996$. We may expect similar small divergences in the present case, especially since the observed lines can be in error by a tenth metre. The lines $\lambda = 5693, 4466$ may be used to determine the constants in a Rydberg formula, and there results

$$\lambda = 3646{\cdot}59 \frac{(m + 0{\cdot}0036)^2}{(m + 0{\cdot}0036)^2 - 9},$$

with a limit even closer to Balmer's limit than that of the Sharp series. A change of one-tenth metre in the first line makes a much smaller value of μ.

This formula gives no other lines falling in the observed range of the Wolf-Rayet spectrum, except such as coincide with the more usual hydrogen lines, so that the existence of the series cannot be tested further. It is a special case of the two series

$$\lambda = \lambda_0 \frac{(m + \frac{1}{3})^2}{(m + \frac{1}{3})^2 - 4}, \qquad \lambda = \lambda_0 \frac{(m - \frac{1}{3})^2}{(m - \frac{1}{3})^2 - 4},$$

where $m = 3$ in the first series, and $m = 5$ in the second, give the two lines. The leading members of the first series

$$\lambda = 3646{\cdot}5 \frac{(m + \frac{1}{3})^2}{(m + \frac{1}{3})^2 - 4}$$

are

$m = 3$	$m = 4$	$m = 5$	$m = 6$
$\lambda = 5695$	$\lambda = 4633{\cdot}8$	$\lambda = 4243$	$\lambda = 4050{\cdot}6$,

and the second of these is again very close to the observed centre $\lambda = 4636$ of a bright band in the Wolf-Rayet stars—a band without any identification at present.

But a more convincing argument for its origin from hydrogen is found in an examination of the stars of types O*a*, O*b*, O*c* in the

Harvard classification.* The bright lines in these stars are H_α, H_β, H_γ, H_δ, H_ϵ, the Pickering lines H_γ', H_δ', H_ϵ', and additional lines at $\lambda = 4633$, 4688, 4472, 4059. The third is weak, and due to helium. The second is the Rydberg line of hydrogen. These stars apparently, with the exception of a weak helium line, show mainly hydrogen, and it is a tempting hypothesis to suppose that λ 4633 and λ 4059 are due to hydrogen in an unusual condition.

The accuracy of determination of the Wolf-Rayet lines does not appear to preclude the identity of λ 4636 and λ 4633.

The leading members of the second series

$$\lambda = 3646{\cdot}5\,\frac{(m-\frac{1}{3})^2}{(m-\frac{1}{3})^2-4}$$

are

$\lambda = 4$	$m = 5$	$m = 6$	$m = 7$
$\lambda = 5191$	$\lambda = 4466$	$\lambda = 4165{\cdot}6$	$\lambda = 4007{\cdot}4$

The first has not been found, and the second has been discussed. If a more extended investigation of the spectrum should reveal the first, the argument would be greatly strengthened.

All the lines included in

$$\lambda = \lambda_0 \frac{m^2}{m^2-9}, \qquad \lambda = \lambda_0 \frac{m^2}{m^2-36}$$

where m takes integral values, not forming part of the systems of Balmer and Pickering, have now been examined.

The intermediate series

$$\lambda = \lambda_0 \frac{m^2}{m^2-25},$$

where $\lambda = 3646{\cdot}5$, has, for its chief line in the visible spectrum, not included in other series, $\lambda = 4596$ corresponding to $m = 11$. This is also a Wolf-Rayet line of unknown origin, but of somewhat lower intensity than those given in the foregoing list.

It seems probable that all the lines

$$\lambda = \lambda_0 \frac{m^2}{m^2-p^2},$$

where m and p take integral values, are included in the astrophysical spectrum of hydrogen. A more exhaustive study of the Wolf-Rayet spectrum from the experimental side is necessary, and especially a search for weaker lines, before this suggestion can be confirmed. But it supplies a very probable origin for the lines

$$\lambda\lambda\ 5693,\ 4466,\ 4634,\ 4596,$$

which are associated with unusual brightness of the Sharp series of hydrogen.

* Pickering and Cannon, *Harvard Annals*, vols. 28–29, p. 233.

Total Eclipse of the Sun, 1912 October 10. Report on an Expedition to Passa Quatro, Minas Geraes, Brazil. By A. S. Eddington and C. Davidson.

(*Communicated by the Astronomer Royal.*)

An expedition to observe the total solar eclipse of 1912 October 10 having been sanctioned by the Admiralty, the Royal Observatory party proceeded to Passa Quatro, a village about 180 miles by railway from Rio de Janeiro.

The party were much indebted to the Brazilian Government for their hospitality and liberal assistance. The observers were entertained as the guests of the country during their stay; and free conveyance on the railways was provided for them and their instruments. They were also much indebted to Dr. Morize, Director of the Rio Observatory, for the arrangements made for their comfort, and for much general assistance.

Valuable assistance was rendered by Mr. J. J. Atkinson, who went with the expedition from England, and by Mr. O. Couto de Aguirre and Mr. Leslie Andrews, who joined them at Passa Quatro on October 3; they joined in all the work of the expedition, erection and adjustment of the instruments, etc.

Acknowledgments are also due to the Royal Mail Steam Packet Company, who conveyed the instruments and other baggage free of charge.

The party landed at Rio de Janeiro on September 16. Owing to various delays with the heavy baggage, Passa Quatro was not reached until September 22, and it was not possible to make a start with the preparation of the instruments until September 24. On that day brick piers were built by local workmen to carry the cœlostats and heliostat. Three huts, of light wooden frames covered with Willesden waterproof canvas, were taken out from Greenwich and erected. One of these contained the instruments fed by cœlostats; a second contained the spectroscope and heliostat, and was consequently placed with its length nearly at right angles to the first; the third was made light-tight and formed a very satisfactory developing hut.

The charge of the instruments was distributed as follows:—

Thompson coronagraph—Mr. Eddington.

Six-inch refractor with green colour filter—Mr. Aguirre.

Six-inch triplet with green colour filter—Mr. Andrews.

Spectrograph—Mr. Davidson, assisted by Mr. Atkinson.

M. Pierre Seux kindly volunteered to count seconds during totality.

Rehearsals were started on October 7, and repeated at frequent intervals during the next two days.

The weather during the three weeks preceding the eclipse was on the whole very favourable. Rain, however, set in at noon on the day before, and continued without intermission all through

the day of the eclipse. The expedition was thus entirely unsuccessful.

Continued wet weather greatly interfered with the packing, which was not finished until October 16. The observers left Rio for England on October 23, the heavy baggage, which had not then reached Rio, being forwarded by a later boat.*

The programme of observations consisted of three parts:—

(1) Large-scale photographs of the corona to show structural detail.

(2) Photographs of the corona through a green colour filter, to determine the distribution of coronium.

(3) Photographs of the ultra-violet spectrum of the flash and corona with special reference to the region λ 3000—λ 3300.

Large-Scale Photographs of the Corona.

For this purpose the Thompson coronagraph, used in previous expeditions, was taken out. It consists of an object-glass of 9 inches aperture and 8 feet 6 inches focal length in combination with a concave telephoto lens of 4 inches aperture and 16 inches focus, fitted as a secondary magnifier. This gives an image of the Sun 4 inches in diameter, with a field for full pencils of 14 inches. The total length of the instrument is 12 feet, the equivalent focal length being 36 feet. A 16-inch cœlostat was used to reflect the light into the telescope.

The coronagraph was mounted on wooden tripod stands and pointed slightly downwards. A guillotine shutter, wholly separated from the telescope, was set up immediately in front of the dew cap; by means of a cord the exposures could be given by the observer at the camera end. The coronagraph was focussed by means of the image of an object in the plane of the plate reflected from the plane mirror of the cœlostat.

It was arranged that before totality the lengths of the diminishing crescent should be observed on the ground glass of the coronagraph, so that a signal might be given to the observer with the spectroscope to start the photographs of the flash spectrum.

During totality it was proposed to take the following series of photographs:—

Exposure.	Plate.	Size.
2^s	Ordinary	12 in. × 10 in.
5	Empress	15 in. × 15 in.
15	Special Rapid	,, ,,
25	,, ,,	,, ,,
10	Empress	,, ,,
2	Process	12 in. × 10 in.

* A descriptive account of the expedition is given in *The Observatory*, vol. xxxv. p. 62.

About 9 seconds is required, between the end of one exposure and the beginning of the next, for changing the plate-holders.

Photographs with a Green Colour Filter.

A 6-inch telescope of 7 feet focus and a 6-inch Cooke triplet of 27 inches focus were mounted side by side, and fed by a second 16-inch cœlostat. Their supports were independent, so that the operation of changing plates in the one would not shake the other. Each of them was provided with a dense green colour-screen of dyed gelatine cemented between glass plates placed immediately in front of the photographic plate. One of the colour-screens was made by Zeiss, the other was a "mercury green" filter by Wratten & Wainwright; from tests of the transmitted spectrum it appears that the two filters were almost identical. The object was to eliminate as far as possible all light except that proceeding from the coronium radiation λ 5303; the preliminary tests indicated that, having regard to the strength of the green coronium line in previous eclipses, the narrow band of continuous spectrum transmitted by the filter would have only a small effect in comparison with the coronium light. Allochromatic plates were used in preference to panchromatic, as the decline in sensitivity of these plates on the red side of λ 5303 was advantageous in narrowing the transmitted band of continuous spectrum. The telescopes were focussed visually on the planet Venus, with the colour filters in position.

In the programme of observations a wide range of exposures was provided for. It was intended to expose four plates with each instrument for 2, 6, 20, and 60 seconds respectively. Plate-holders, each carrying four plates, were provided for each instrument, so that the operation of changing the plates could be performed very rapidly.

The two telescopes and the Thompson coronagraph with their respective cœlostats were mounted in the same hut. Some care was necessary in calculating the positions of the shadows to fit all the apparatus in together.

The Spectrograph.

The quartz spectroscope, used by Major Hills at the Indian eclipse of 1898, and at Ovar 1900, Sumatra 1901, and Sfax 1905, by Dr. Dyson, was kindly lent by Professor Newall for the expedition.

Its details are:—

Collimator and Camera.—Single quartz lens, 3 inches aperture, 33-inch focus.

Prisms.—Four double quartz prisms of 60° (each prism being composed of two half prisms of right- and left-handed quartz), $3\frac{1}{2}$-inch base, $2\frac{3}{4}$-inch height.

Slit.—2 inches by ·0012 inch.

Prisms at minimum deviation for λ3300.

Extent of spectrum, λ 2960 to λ 4300.

The collimator was fed by a concave mirror of speculum metal of 9 inches aperture and 10 feet 2 inches focus.

This condenser was supplied with light from a heliostat furnished with a 9-inch plane speculum mirror.

Several changes had been introduced in the hope of extending and improving previous results. In former work the lower limit of wave-length attained has been about 3300; particular attention was therefore given to this region and beyond. In order to pass beyond this limit a speculum mirror was substituted in the heliostat for the silver on glass, which is a bad reflector of radiation of shorter wave-lengths. To concentrate the light, the uncorrected quartz condensing lens was replaced by a concave speculum mirror, which, notwithstanding that the angle between the incident and reflected pencils was 3°, gave a perfectly defined solar image on the slit.

In the eclipses of 1898 and 1900 plates had been used in the camera, but the field is very strongly curved, the spectrum coming to a focus on a cylindrical surface whose radius is only 7 inches, and the field of good definition on plane plates is therefore small. In the eclipses of 1901 and 1905 this difficulty had been partially met by using two plates inclined at an angle each covering half the field, but on the present occasion films carried in sheaths bent to the right curvature were to be used instead of plates. In the experimental photographs good definition was obtained over the full length of the film (6½ inches); that is, as the spectroscope was set, from λ 2960 to λ 4300. The absolute limit imposed by the absorption of the atmosphere is about λ 2970.

The concave condenser was mounted 10 feet from the slit and fed from the heliostat placed about 1 foot in front of the slit, in such a manner that the convergent beam just passed the edge of the heliostat mirror, the inclination of the incident to the reflected beam being about 3°.

Father Sidgreaves, to whom our thanks are due, had kindly lent a 7-inch concave for the purpose; but during the course of experiments a 9-inch concave by Sir W. Herschel from the museum of the Royal Observatory was found to be more suitable on account of its greater focal length, which permitted a reduction in the angle between the incident and reflected beams. The 9-inch flat had been ordered in the meantime to feed the 7-inch, but it was nevertheless decided to use the 9-inch as preferable.

The spectroscope was mounted on a table inclined at an angle of 20° to the horizontal, so that the slit should be tangent at the point of second contact. The solar image was set on the slit and viewed with a low-power microscope inclined at a small angle.

The programme which it was hoped to carry through was as follows:—

Following Professor Fowler's suggestion that observations of the chromosphere could be made during the partial phase, it was intended to photograph the spectrum close to one of the cusps,

commencing 15 minutes before totality. Exposures of 2 minutes were to be made up to about 2 minutes before second contact. This part of the programme was purely experimental; the conditions were not altogether favourable for testing Professor Fowler's method, as the slit was set at the best angle for the flash spectrum, and could not be altered to suit the cusps. The film having been changed, exposures of 3 seconds with the Sun's limb tangential to the slit were to be given from 15 seconds before totality until totality was complete. The film would then be displaced once more and exposed at the same point for the corona for as long as totality lasted. No attempt was to be made on the third contact, but another series of exposures were to be made on a cusp until 15 minutes after totality.

Our thanks are due to Mr. Knobel, who took great interest and trouble in having experiments made to determine the most suitable emulsion for photography in the ultra-violet region. It appeared that the slowest plates in the ordinary spectrum were as fast or faster than the most rapid when used for the extreme ultra-violet. Special films made for him were taken to Brazil, and it was intended to use one for one of the series of exposures on the cusp.

The Variable Star R Cygni.

By E. E. Barnard.

At the request of Rev. T. E. Espin, I examined R Cygni at its minimum in September 1912 with the 40-inch telescope for the nebulosity which had been reported to have been seen around it at the time of minimum.

Under the best conditions there was no evidence of any nebulosity about the star or in its immediate vicinity.

During these observations the variable was measured with respect to θ Cygni for position, as was also the nearby star B.D. $+49^{\circ}3065$.

θ and R Cygni.

1912 Sept. 17	$\Delta\alpha + 0^{m}\ 22^{s}\cdot43$ (10 tr)	$\Delta\delta - 0'$	$56''\cdot6$ (4)
22 $\Delta\alpha \cos\delta$ $217''\cdot0$ (5)	$+0\ \ 22\ \cdot51$	-0	$55\ \cdot9$ (5)
	$\Delta\alpha + 0^{m}\ 22^{s}\cdot47$	$\Delta\delta - 0'$	$56''\cdot2$

With the position and proper motion of θ Cygni from Boss's Preliminary General Catalogue, the following position results for R Cygni:—

$1912\cdot0 \qquad \alpha\ 19^{h}\ 34^{m}\ 27^{s}\cdot40 \qquad \delta + 50^{\circ}\ 0'\ 4''\cdot0.$

This differs from the place of the variable from the General Catalogue by $-0^{s}\cdot05$ and $-0''\cdot9$.

The proper motion assigned by Boss is $-0^{s}\cdot0009$ and $-0''\cdot011$.

The following measures of +49°3065 were also made with respect to θ Cygni on September 22 :—

$\Delta\alpha \cos\delta$ 239″·2 (4)	$\Delta\alpha$ +0^m 24^s·81	$\Delta\delta$ +0′ 32″·7 (4)
	+0 24 ·85 (10 tr)	
	$\Delta\alpha$ +0^m 24^s·83	$\Delta\delta$ +0′ 32″·7

This gives the position for +49°3065 :

1912·0 α 19^h 34^m 29^s·74 δ +50° 1′ 32″·9.

The variable was measured on three nights with respect to the above star (+49°3065) :

1912 Sept. 22	193°·65	91″·12
29	193 ·87	91 ·05
Oct. 1	193 ·92	91 ·11
	193°·81	91″·09

The faint star No. 5 was measured on one night with respect to R Cygni :

1912 Oct. 15 108°·4 14″·63 15th magnitude.

The small star was very faint from bad seeing, so that the measure is not likely to be very good.

I have thought that the accompanying diagram might be of service in the ready identification of the variable when in its fainter

Region of R Cygni.

phases, as it is easy to mistake it for one of several small stars near. The stars Nos. 1, 2, 3, 4, and 6 are roughly located from a sketch. I regret that other work prevented their exact measurement. They have been measured, however, by Mr. Espin (see *M.N.*, vol. lxxiii. p. 164).

The following magnitudes were roughly assigned to the stars on the sketch referred to :—

1 = 12 mag.
2 = 13·3
3 = 13·7
4 = 13·7
5 = 15
6 = 13

On the same sketch (Sept. 17) the variable was marked $12\frac{1}{2}$ magnitude.

The following are the notes:—

1912 Sept. 17^d 10^h 40^m.—R Cygni when compared with B.D.+49°3065 showed no difference of focus. Star 2, however, looked somewhat ill defined and showed a focus 2^{mm} longer than for R, but the seeing was too poor to be certain. R was 1 magnitude brighter than Star 2, and $\frac{3}{4}$ mag. less than Star 1.

Sept. 29^d 10^h 40^m.—R Cygni was the mean brightness of Stars 1 and 2. Sky thick and white.

Oct. 1^d 8^h 15^m.—The variable was $\frac{2}{10}$ mag. less than Star 1, and about the mean magnitude of Stars 1 and 2. Seeing good. R Cygni was clearly defined, with no trace of nebulosity. Star 2 seemed ill defined. It was not so well defined as the variable.

Oct. 15^d 8^h 0^m.—It was $\frac{1}{10}$ magnitude less than Star 1—almost exactly equal to that star. It was 1 magnitude brighter than Star 2.

See recent papers on this star by H. D. Curtis, *M.N.*, vol. lxxiii., December 1912, p. 92, and by Rev. T. E. Espin, *M.N.*, vol. lxxiii., January 1913, p. 165.

Yerkes Observatory, Williams Bay, Wisconsin:
1913 *March* 1.

Notes on the Nebula H.I.159 Cassiopeiæ.

By Mrs. Isaac Roberts.

(*Communicated by the Secretaries.*)

In reply to the question raised by me in one of the last paragraphs of my paper on the Nebula H.I.159 Cassiopeiæ, *M.N.*, 1912, page 413, as to the structure of p_1, p_2, p_3, an admirable photograph was secured by Astronomer Francis G. Pease with the 60-inch reflector of the Mount Wilson Solar Observatory, on 1912 November 8, with an exposure of four hours, on a Seed 23 plate, two prints of which have been kindly sent me.

In the letter accompanying these prints, Mr. F. G. Pease writes that "two well-separated stars are in each of the positions indicated." "These are shown on the enclosed photographs, p_1 standing out very distinctly, while the fainter components of p_2 and p_3 are almost lost."

I have compared the photograph of the region of H.I.159 obtained with the Mount Wilson 60-inch reflector with the Isaac Roberts plates 1727 and 3197, secured on 1894 November 18 and

1904 January 9 with the 20-inch reflector and with exposures of 30 and 90 minutes respectively. The following is the result of my comparison:—

The scale of the I.R. negatives is one-third that of the Mount Wilson reflector photograph; the scale of the drawing printed in the *M.N.*, vol. lxxii., plate 3, is 12 times that of the I.R. negatives.

The nebulous condensations in H.I.159 measured by me on the I.R. plates 1727 and 3197 are seen with more details on the photograph taken by Mr. F. G. Pease.

The condensations 3 which appeared with multiple nuclei on the I.R. plates are shown on the F.G.P. photograph to be formed by a group of very close condensations.

The faint ray directed northward and in the vicinity of the nebulous knots 3 and 2 is shown on the F.G.P. photograph to be formed by a very small star and a streak of very faint nebulosity *sf*, well separated, both involved in fainter nebulosity.

The condensations 4, 5, 6, 8 on the I.R. negatives and the F.G.P. photograph are involved in very dense nebulosity. Other condensations are visible in the spiral branch of H.I.159 on the F.G.P. photograph. The condensations 7 and 9 of a star-like nature on the I.R. plates and the F.G.P. photograph are free from denser nebulosity on their *f* side.

The denser part of the Nebula H.I.159 formed by one or more left-hand spiral branches is surrounded by more diffused nebulosity which is only very faintly seen on the I.R. plate 1727 and better seen on I.R.3197. On the F.G.P. photograph this diffused nebulosity is very conspicuous; in its southern region it contains diffused nebulous masses which are not distinctly seen on the I.R. plates.

The stars in the immediate vicinity of the Nebula H.I.159 are more numerous on the F.G.P. photograph than on the I.R. plates and are admirably defined.

The patch p_1, which appears of nebulous aspect on the I.R. plates, is formed as stated above, by two small stars well separated on the F.G.P. photograph; the smaller star north preceding the double star p_1, well defined on the F.G.P. photograph, appears on the I.R. plate 3197 like an extremely faint nebulous knot.

The patches p_2 and p_3, which appear nebulous on the I.R. plates 1727 and 3197, are shown to be well-defined stars on the F.G.P. photograph, "with fainter components which are almost lost" on the print.

Many of the stars measured by me in the immediate vicinity of the Nebula H.I.159 appear on the F.G.P. photograph with well-defined components. Thus:

Star 1, elongated on I.R.3197, appears on the F.G.P. photograph with 2 fainter stars in its *nf* region.

Star 2, which on the I.R. plates presents a nebulous knot on its *np* side, is shown to be a double star well separated on the F.G.P. photograph.

Star 6. The group of smaller stars encircling star 6 on its *s* and *f* sides, well seen on the F.G.P. photograph, appear like nebulous knots on I.R.3197; they are extremely faint on I.R.1727.

Star 7, elongated on I.R.3197 and with traces of nebulosity on its *p* side on I.R.1727, is shown to be a double star on the F.G.P. photograph; the component on its *sp* side is very small.

Stars 8 and 9, which appear with nebulous outlines, elongated *sp* to *nf* on I.R.3197 and with traces of extremely faint nebulosity on I.R.1727, are shown to be the 2 brighter stars of a group of six stars.

Star 10. On the *sp* side of star 10, on I.R.3197, are very faint traces of nebulous knots which are shown to be 2 well-defined stars on the F.G.P. photograph.

Star 11 presents in its *np* region 2 very small stars which are not visible on the I.R. plates 3197 and 1727.

Star 12, dense and irregular on I.R.3197 but well-defined on I.R.1727, presents on the F.G.P. photograph a small companion on its *sf* side.

Star 13, dense and diffused on I.R.3197 but better defined on I.R.1727, presents on the F.G.P. photograph a small companion on its *nf* side. The group 14, 15, 16, which is extremely faint on I.R.3197 and 1727, is well defined on the F.G.P. photograph.

Star 14 on the F.G.P. photograph is preceded by a slightly brighter star which appears of nebulous aspect on the I.R. plate 1727, and is followed in its northern region by a much fainter star invisible on the I.R. plates.

Star 16, which appears of a nebulous aspect on I.R.1727, presents on the F.G.P. photograph 2 other stars in its vicinity.

Star 18, which appears with faint traces of nebulosity on its *sp* side on I.R.1727, is shown to be a double star on the F.G.P. photograph; the small companion is in the *sp* region of the principal.

Star 19, diffused an I.R. 3197 but better defined on I.R.1727, also presents a small companion on its *sf* side, as is shown by the beautiful photograph obtained at the Mount Wilson Solar Observatory.

In conclusion, many of the objects which appear of a nebulous nature on the Isaac Roberts negatives are really stellar, and at the limit of visibility with the 20-inch reflector.

In the great progress achieved to-day in the field of nebular photography under the wonderfully clear sky of California and with the powerful instruments constructed and guided by most able hands, the late Isaac Roberts would have been, no doubt, the first to rejoice.

Maison de Rosa Bonheur, By Thomery:
1913 *February* 8.

Observations of Gale's Comet made at the Sydney Observatory.

(*Communicated by W. Ernest Cooke, M.A., Government Astronomer.*)

Various observations of Gale's Comet were made at the Sydney Observatory, but the only ones worth recording are the measures of inclination of the two principal tails to one another. These two tails were of almost equal magnitude visually, but on the photographs one of them is very short and brushy, whilst the other is much longer, and more distinct. The lens was a 6-inch Dallmeyer of three feet focus, mounted on the astrographic telescope. The photographs of September 15, 16, and 19 are fairly distinct, and the measures for those dates are considered pretty good, but the later ones are faint and rather uncertain. On the whole there seems to be evidence of a gradual separation.

1912 September 15	44°
16	45
19	48½
20	53
21	46
28	53

1913 *January* 22.

Notes on Fireballs and Shooting Stars. By W. F. Denning.

I have been comparing and analysing certain results of meteoric observations obtained here and at other places in past years.

The Principal Showers.—With regard to the heights, lengths of paths, and velocities of meteors belonging to some of the principal annual showers,* I have found the following mean values from my computations during the last twenty-seven years. For most of the systems the number of meteors utilised is perhaps too small to yield very dependable averages, but in the instances of Perseids and Leonids the results are ample. The Leonids are usually about two miles higher than the Perseids, owing probably to their greater velocity, the observed rates being 47·9 and 38·8 miles per second respectively, which is near the theoretical values 44 and 39 miles.

* There are insufficient determinations of the real paths of the May Aquarids, October Orionids, and November Andromedids to warrant their inclusion in the table.

Date.	Name of Shower.	Radiant Point.		No. of Meteors.	Height at Beginning.	Height at Ending.	Length of Path.	Velocity per Second.	No. of Meteors for Velocity.
					miles	miles	miles	miles	
Jan. 1–4	Boötids	231°	+52°	8	63·0	50·0	65·0	23·0	8
April 18–24	Lyrids	271	+33	7	82·3	51·4	57·0	32·7	6
April	Virginids	209	−8	7	61·6	48·1	59·7	18·4	5
June–July 8	Scorpiids	252	−22	7	59·1	42·7	97·9	16·0	6
July 15–Aug. 1	Capricornids	305	−11	10	60·9	44·7	44·2	15·0	7
July–Aug.	Perseids	45	+57	76	81·2	53·4	47·8	38·8	43
July 28–Aug. 11	Aquarids	339	−11	7	61·6	47·0	66·1	18·6	5
September	β Piscids	346	+3	6	72·3	38·3	82·8	19·2	3
October	Arietids	Several		7	78·0	37·4	75·9	21·2	5
November	Taurids	Several		18	71·1	43·4	61·7	24·0	17
Nov. 12–17	Leonids	151	+23	34	82·7	55·7	54·1	47·9	21
Dec. 9–16	Geminids	110	+33	9	75·5	45·5	59·6	24·6	6

Slow meteors from radiants low in the English sky, like the Virginids, Scorpiids, Capricornids, Aquarids, etc., with apparent radiants preceding the apex of the Earth's way about 90°, usually have heights of from 67 to 44 miles, and rather long flights of 60 to 100 miles.

The slow meteors with tolerably high radiants often penetrate the atmosphere until within 25 to 30 miles of the Earth's surface. Swift-streaking meteors like Perseids and Leonids very rarely descend lower than 45 miles.

There is a decrease in height almost proportionate with decrease in velocity, and this appears to be at the approximate rate of $1\frac{1}{2}$ miles for every 2 miles per second in speed. But meteors appear under different conditions which have an influence on their height, length, and velocity; the varying altitude of the radiant point is an important factor in introducing discordances.

Mean Heights of Meteors.—The mean heights, etc., of 550 fireballs and shooting stars computed by me and observed in the United Kingdom since 1886 are as under. For the elevation and length of paths the whole number could be used, but for the velocity the mean is derived from 397 meteors, the remainder lacking suitable observations:—

Height at appearance	72·1 miles.
„ disappearance . . .	46·3 „
Length of path	60·4 „
Velocity per second	27·3 „

All sorts of meteors are comprised in this result.

I think it is rare for the height of a meteor, when it first becomes visible to the eye, to exceed 100 miles, and I quite agree with Lieut.-Col. Tupman in this conclusion. There are some

pretty well-supported instances of this height, and I have myself obtained it in several cases; but the more one has to discuss data of this kind, the more forcible the impression grows that such abnormal elevations are due to observational errors. The exact beginning of a meteor is seldom well seen, if seen at all, and the person recording it often puts it back too far in the line of flight, and a very few degrees will sometimes make a great addition to the height. Considerable differences are occasionally found between the observed and theoretical velocities. I have seen meteors which certainly exhibited a speed of between 50 and 100 miles per second, if not more. They were recorded by other observers as well, and even after allowing the widest limits for mistakes in estimating the duration of flight, the motion must have been very exceptional. The fireball of 1902 September 25 may be cited as furnishing an example, for I computed its observed speed as more than 100 miles per second. The meteor was directed from a radiant at $255^\circ + 46^\circ$, and it is curious that both swift and slow objects diverged apparently from the same position! A similar peculiarity was noticed in regard to a shower witnessed by me on the mornings of 1901 August 10 to 20 from $106^\circ + 52^\circ$. Several of the meteors were of the swift, streak-leaving class, but one of them was very slow and streakless. Fortunately, it also came under the accurate eye of the late Professor A. S. Herschel, and the velocity from our careful estimates worked out at 15 miles per second, whereas the parabolic speed was 34 miles per second for a meteor with a radiant in the position assigned in the Lynx. When the radiant of a meteor is low, its long and nearly horizontal flight is often unduly slow. Thus the Perseids of 1908 Aug. 2, $10^h\ 26^m$, and 1900 Aug. 10, $10^h\ 6^m$, had velocities of 25 and 27 miles per second respectively instead of 39; while the Leonids of 1910 Nov. 16, $12^h\ 19^m$ and 1908 Nov. 17, $11^h\ 9\frac{1}{4}^m$ moved at a rate of 30 miles per second instead of 44. Many other similar cases could be quoted.

Errors in Real Path Determinations.—With a view to find the probable error in computations of meteor-heights, I recently compared 75 real paths of the same meteors as derived by Professor A. S. Herschel and by myself in the years chiefly from 1900 to 1904. Our methods were different and perfectly independent, and I found the following mean discordances:—

At Beginning.	At Ending.	Length of Path.	Velocity per Second.
4 miles	$3\frac{1}{2}$ miles	$4\frac{3}{4}$ miles	$3\frac{1}{2}$ miles

Rejecting 10 cases where the differences were greatest, the discordance is reduced about 1 mile for each of the above features. It appears certain that for a meteor really well observed at two places the probable differences in the computed elevation would not exceed 2 miles.

Variation in Hourly Numbers.—As regards fireballs, I have tabulated the times of appearance of 1734 recorded during the last half a century, and they appear to afford evidence of a 2-hour

period of frequency. Thus between 4^h and 5^h, 6^h and 7^h, 8^h and 9^h, and 10^h and 11^h the numbers showed a marked excess relatively to the intervening hours. After midnight there is deficient material, and the numbers exhibit a great decline, resulting from the lack of observers. For the earlier hours between 4^h and 5^h and 6^h and 7^h during a great part of the year there is daylight, and no observations are possible at these periods; but if fair allowances are made, it becomes obvious that, though few fireballs were recorded at the hours named, they must still have been plentiful relatively to the scanty opportunities afforded for them to be seen. And it must be remembered in this connection that no Perseids or Leonids are visible in the early hours of the evening, and that the number of really large meteors recorded after, say, 8 p.m. has been greatly augmented by the fact that regular observers often commence to watch at that hour or soon afterwards. The particular hour which appears to have been most thoroughly utilised for meteoric work is that from 10 to 11 p.m.

The following are the hourly number of fireballs observed. I have added the number of nights annually when the Sun set more than half an hour before the middle time of the hour, and the last column shows the computed relative number on this basis.

Hour.	Fireballs observed.	Nights when sunset occurred 30 min. before middle time of the hour.	Computed relative number of Fireballs.
4 to 5	35	39	327·6
5 to 6	76	112	247·7
6 to 7	146	174	306·2
7 to 8	177	236	273·8
8 to 9	319	308	378·0
9 to 10	318	365	318·0
10 to 11	428	365	428·0
11 to 12	235	365	235·0

The great increase in abundance between 8 and 9 and 10 and 11 is remarkable. The hour from 4 to 5 is never free from twilight, and large allowance must be made for this and other disadvantages. It is true that a really brilliant fireball would often arrest attention in a light sky, but not with the same frequency as on a dark night, for it is the luminous effect produced by the object which incites observers to glance upwards and discover the source.

Quite possibly the horary variation is only accidental, but it is corroborated by my observations of shooting stars. I have not, however, always kept a register of the number visible each hour, but from a portion of my results have derived the following rates per hour. They are not affirmed by the older researches of Coulvier Gravier in France from 12 years' observations (1846-57) whose hourly rates I have placed alongside my own for ready comparison. Coulvier Gravier's figures indicate the highest

numbers during those hours which are my minima, so the idea of any anomalous irregularity affecting the horary rates must be abandoned. The mean values from my own and the French deductions probably represent the correct hourly numbers visible within very small limits of error.

Hour.	W.F.D.	Coulvier Gravier.	Mean.
5 to 6	...	7·2	...
6 to 7	8·4	6·5	7·5
7 to 8	7·2	7·0	7·1
8 to 9	8·3	6·3	7·3
9 to 10	8·4	7·9	8·1
10 to 11	10·4	8·0	9·2
11 to 12	10·4	9·5	10·0
12 to 13	11·4	10·7	11·0
13 to 14	12·8	13·1	13·0
14 to 15	13·5	16·8	15·2
15 to 16	14·5	15·6	15·1
16 to 17	13·2	13·8	13·5
17 to 18	11·4	13·7	12·6
18 to 19	...	13·0	...

The general maximum of meteoric appearances occurs between 14 and 16^h. For the morning hours Coulvier Gravier's figures exceed mine, but I believe he included the Perseids, whereas I excluded them, my intention being to find the values for ordinary nights of the year when no very special displays unduly swell the numbers.

We should naturally expect the maximum to occur when the point towards which the Earth is moving is on the meridian at about 18^h, but there is a pretty rich congregation of streams far west of this and I believe this operates in developing the greatest richness at about 15^h. I gave some particulars regarding these westerly streams in a paper entitled "Fireball Radiants" published in *Monthly Notices*, lvii. 361–3.

With regard to the streaks or trains left by certain large meteors, I have been trying, on the suggestion of Professor C. C. Trowbridge of the Columbia University, New York, to ascertain the velocities and directions of their drifts in our upper atmosphere. Their altitude above the Earth's surface is usually 65 to 50 miles, and they are frequently generated either by the Perseids or Leonids, or meteors of that class. The observations in past years have been, for the most part, very imperfect and inaccurate, and the results generally derived from them can only be regarded as rough approximations. In a few cases, however, such as those relating to the fireball of 1894 Aug. 26 and 1909 Feb. 22, the data are very full and reliable. Of 19 long-enduring streaks, I obtained the mean motion slightly exceeding 2 miles per minute,*

* In certain other instances the motion was decidedly greater, ranging between 200 and 400miles per hour!

and, comparing this with certain other planetary velocities and giving the horary rates, we have :—

1866–1910.	Fireball streaks 124 miles per hour, direction usually to S.E. or E., height 65 to 50 miles.
1860 Feb.–May.	New belt formed on Jupiter, N. of equator, material spread 170 miles per hour to E.
1880–1 Oct.–Jan.	New belt on Jupiter, N. of equator, 112 miles per hour.
1883 Autumn.	Dust from Krakatoa circulated around Earth 3 times in 30 days = 104 miles per hour at equator. It moved 92 miles per hour at the later stages—Prof. K. I. Kiessling. 83 miles per hour to W., 100 miles high—J. E. Clark. 80 miles per hour to W., 100 miles high—S. E. Bishop.
1880–1881.	Rate of equatorial spots on Jupiter relatively to red spot in S. hemisphere, 265 miles per hour to E.
1898–1907.	The same features, 250 miles per hour.
1876 and 1903.	Motion of equatorial spot on Saturn (1876) relatively to spots in N. temp. region of planet (1903), 920 miles per hour to E.

Bristol:
1913 *March* 7.

Errata in Monthly Notices, vol. lxix.

Page 635, line 7,

for $(2n+j-1)$ *read* $(n+2j+2i)$.

Page 637, line 2,

for $p_i = \frac{b_{i,1}}{a^i}$ *read* $p_i = \frac{b_{i,1}}{ab_{i-1,1}}$,

as on pp. 640–641.

Page 647, line 5 from bottom,

for 2 !, 3 !, etc. *read* 1/2 !, 1/3 !, etc.